Downward Nominal Wage Rigidity and Inflation Dynamics during and after the Great Recession

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This version: February 21, 2018

Abstract

I develop a New Keynesian dynamic stochastic general equilibrium model with heterogeneous workers whose wage settings are subject to downward nominal wage rigidity (DNWR), to address two seemingly puzzling features in the inflation dynamics in recent years: *the missing deflation* during the Great Recession and *the excessive disinflation* in the subsequent years. I demonstrate that DNWR creates a time-varying wedge between the output gap and the marginal cost of producing one unit of output, which in turn appears in the Phillips curve as a shift parameter. The model's dynamics are asymmetric and non-linear, depending on the evolution of cross-sectional wage distribution. During a severe recession, in particular, a larger fraction of workers being constrained by DNWR raises the wedge sharply and it makes the observed Phillips curve flatter. In addition, the presence of the zero lower bound of the nominal interest rate reinforces the mechanism by amplifying the impacts of an exogenous shock. I calibrate the model to match the wage distribution in U.S. data to find that it successfully replicates the moderate decline of inflation and the large contraction of real quantities during the Great Recession. The endogenous shifts of cross-sectional wage distribution account for the slow recovery of inflation in the subsequent periods.

JEL classification: E31, E24, E52.

Keywords: DNWR, cross-sectional wage distribution, missing deflation puzzle, excessive disinflation puzzle, Great Recession, zero lower bound of nominal interest rate.

*I am deeply grateful to Susanto Basu, Pablo A. Guerrón-Quintana, and Dongho Song for their invaluable guidance and support throughout the project. I thank Ryan Chahrour, Peter Ireland, Rosen Valchev, and all the participants at the Boston College Macroeconomics Lunch Seminar for their helpful comments and suggestions. Department of Economics, Boston College, 140 Commonwealth Avenue, Maloney Hall, Chestnut Hill, MA 02467. Email: mineyama@bc.edu

1 Introduction

During the financial crisis of 2008−2009 and its aftermath, the U.S. economy experienced little decline of inflation while suffering from severe economic downturn. The fact is known as the missing deflation puzzle. To this end, Hall (2011), in his Presidential Address to the American Economic Association, argue that the little response of inflation to the long-lasting slack after the Great Recession is not consistent with most of economic theories. Several years later, the recovery of inflation was so slow despite the sluggish but steady improvement of real economic activities in the subsequent years. The shortfall of inflation from 2 percent without many of adverse factors is expressed as a mystery by the Chair of the Federal Reserve in Yellen (2017). Constâncio (2015) labels the phenomenon as excessive disinflation.[1] These observations call into question one of the fundamental theories in modern monetary economics: the Phillips curve relationship between inflation and the level of economic activity.

In this paper, I argue that downward nominal wage rigidity (DNWR) can reconcile both of the missing deflation during the Great Recession and the excessive disinflation in the subsequent years. Specifically, I introduce DNWR for individual workers into an otherwise standard New Keynesian dynamic stochastic general equilibrium (DSGE) model that embeds monopolistically competitive firms with nominal price rigidity and the Taylor (1993)-type nominal interest rate feedback rule. To the best of my knowledge, this is the first study to build a DSGE model with both nominal price rigidity and the explicit constraint on downward nominal wage adjustment for individual workers. In this setting, I demonstrate that DNWR accounts for the flattening of the observed Phillips curve during recessions. Taking into account heterogeneity of individual workers' wages enables the model to replicate many dimensions of non-linearities in the data through endogenous shifts of cross-sectional wage distribution upon exogenous shocks. Moreover, compared to previous studies on DNWR, I find that nominal price rigidity and DNWR have an important interaction to generate substantial persistence of real wage especially downward. Consequently, the model can quantitatively match the key moments of inflation dynamics during and after the Great Recession under plausible parameter values that are consistent with micro evidence. I investigate the role of the zero lower bound of the nominal interest rate as well.

The model is motivated by two empirical facts. First, numerous studies have pointed out that the Phillips curve relationship between inflation and the output gap was altered after the Great Recession (Stock and Watson (2010), Ball and Mazumder (2011), Coibion and Gorodnichenko (2015), etc.). However, as I document in Section 3, the marginal cost representation of the Phillips curve, which is directly derived from firms' price setting behavior, remained stable in the data. Instead, the relationship between the output gap and marginal cost is found to be non-linear in the sense that marginal cost is less responsive to the output gap in recessions. The stylized fact implies that a puzzle indeed lies in the relationship between the output gap and marginal

[1] Constâncio (2015) points out that the excessive disinflation is a common feature of inflation dynamics in advanced economics in recent years.

cost rather than in the Phillips curve itself. Therefore, I focus on the relationship between them to explain the changes of the observed Phillips curve. Second, the rapidly growing empirical literature using micro data uncovers the severity of DNWR during the Great Recession and its aftermath. For instance, previous studies report that the fraction of workers with zero nominal wage changes substantially increased in the periods, which is consistent with predictions of DNWR (Daly and Hobijn (2014), Fallick et al. (2016)). I incorporate these empirical findings into a general equilibrium model to study their aggregate implications, especially on inflation dynamics.

The key mechanism of the model for generating the missing deflation is as follows. DNWR creates a time-varying wedge between real wage and the marginal rate of substitution between consumption and hours worked by impeding wage adjustment to its desired level. The wedge in turn appears in the output gap representation of the Phillips curve as a shift parameter, and it accounts for the flattening of the observed Phillips curve in recessions. Intuitively, the binding DNWR constraint upon a contractionary shock prevents firms' marginal cost from declining. Then, since the New Keynesian Phillips Curve (NKPC) implies that inflation can be expressed as infinite sum of the discounted values of the current and future marginal costs, the dampened response of marginal cost results in little decline of inflation in recessions. On the other hand, imperfect adjustments of price variables are compensated by large contractions of real quantities including the output gap in general equilibrium. As a consequence, even though the marginal cost representation of the NKPC remains unchanged, the observed Phillips curve relationship between inflation and the output gap becomes flatter in recessions in the presence of DNWR.

I allow for heterogeneity of individual workers' wages that might be subject to the DNWR constraint. By doing so, the aggregate dynamics of the model, including the degree of the aggregate wage and price stickiness, crucially depend on the evolution of cross-sectional wage distribution. To be precise, the responses of the model are asymmetric depending on the sign of an exogenous shock. A larger fraction of workers is constrained by DNWR upon a contractionary shock, whereas the constraint comes to bind for a fewer workers upon an expansionary one. Hence, the aggregate wage is more rigid downward than upward and that spills over asymmetry of other variables. The aggregate dynamics are also affected by the size of an exogenous shock, because a larger shock changes the fraction of workers with or without the binding constraint more drastically. Therefore, the mechanism of the missing deflation described above is particularly strong for a large and contractionary shock such as the Great Recession.

It is noteworthy that the mechanism is reinforced by the presence of the zero lower bound (ZLB) of the nominal interest rate. In line with the existing literature such as Christiano et al. (2011), the impacts of a demand side shock are amplified under the ZLB due to the lack of the offsetting monetary policy responses. However, since downward wage adjustment is restricted by DNWR in my model, the amplification effect of the ZLB is exclusively absorbed by further contractions of real quantities without generating a large drop of inflation. The finding is in stark contrast to previous studies such as Gust et al. (2017), who find the responses of inflation as well as those of quantities upon a demand shock are enlarged at the ZLB. This effect helps to reconcile the moderate decline of inflation and the sharp fall of real quantities during the Great Recession.

On the other hand, I find the state dependency of DNWR to be the key feature of the model to address the excessive disinflation after the Great Recession. Since the DNWR constraint should hold in terms of the level of wages, once workers' desired wages fall short of their actual ones upon a contractionary shock, workers never react to improvements of the state of the economy until their desired wages exceed their actual ones. Though this pent-up wage deflation mechanism is mentioned in several studies (Daly and Hobijn (2015), Constâncio (2015), a blog post by Krugman (2014)), I demonstrate its formal link to the excessive disinflation, and derive quantitative outcomes in a framework of a DSGE model.

For quantifying the implications of the model, I overcome two potential challenges in numerical methods. First challenge is computing equilibrium of the model with heterogeneous agents. Since the main focus of this paper is on inflation dynamics at business cycle frequency, I choose to solve the model under aggregate uncertainty by applying the Krusell and Smith (1998) algorithm. Although their original algorithm requires aggregate jump variables to have a closed form solution in terms of aggregate state variables, that condition is not satisfied in my New Keynesian setting with the ZLB. To address the problem, I propose a modified algorithm in which each of the aggregate and individual part of the economy is solved recursively with a global method.

Another important challenge is the parameterization of the degree of price and wage rigidities. It is widely recognized that an estimated DSGE model often identifies much higher parameter values for the degree of price stickiness than the ones implied by micro evidence (Altig et al. (2011), Del Negro et al. (2015)), and the inflation persistence in the model heavily relies on that parameter value. Similarly, Schmitt-Grohé and Uribe (2016) discuss that the parameter governing the degree of DNWR is crucial to quantitative results. To this regard, I calibrate the model to match moments of the individual price and wage changes in U.S. data, and find that my model endogenously generates strong persistence of inflation under the micro founded parameter values.

My quantitative results are summarized as follows. A counterfactual analysis in the calibrated model predicts that the contractionary shock that has the same magnitude as the Great Recession only leads to 2.1-2.4 percentage point decline of the year-on-year inflation rate under plausible assumptions on monetary policy rules. The quantitative result is comparable to the data during the period when the actual inflation rate in the GDP deflator declined by 2.3 percentage point from the peak to the bottom.[2] Regarding the excessive disinflation, the calibrated model suggests the recovery of inflation from a severe recession state that corresponds to the Great Recession is three times as slow as from the median state. For comparison, I show in Section 3 that a stylized New Keynesian model predicts a massive deflation upon the Great Recession shock and a relatively quick recovery from it. It is notable that the only extension of my model from the stylized New Keynesian model is the presence of DNWR and the ZLB, and my model successfully matches the key moments of inflation dynamics during and after the Great Recession.

[2]The peak of the GDP deflator around the Great Recession is 2.5 percent as of 2007Q4, whereas the bottom is 0.2 percent as of 2009Q3.

2 Related literature

This paper falls into the growing literature that studies the missing deflation and the excessive disinflation after the Great Recession. There are mainly two strands of literature regarding the missing deflation puzzle. The first strand emphasizes the importance of the formation of the inflation expectations. Bernanke (2010) suggests that the credibility of modern central banks succeeded in convincing people that extremely high or low inflation would not occur, and this anchored expectation stabilized the actual inflation. In contrast, Coibion and Gorodnichenko (2015) argue that the stability of the inflation expectations is not enough for resolving the puzzle quantitatively. They instead claim that the rises of the household inflation expectations due to the surging commodity prices after 2008 prevented deflation. Bianchi and Melosi (2017) propose another channel through which fiscal and monetary policy uncertainty, that is, a possibility of switching to a high inflation regime driven by large fiscal deficit, keeps the inflation expectations high enough. Del Negro et al. (2015) reconcile these views by estimating a medium scale DSGE model that embeds the financial friction of Bernanke et al. (1999) on the Smets and Wouters (2007) model. They conclude that the anchored expectation view is plausible if the Phillips curve is sufficiently flat, because monetary policy can have strong real effects to stabilize the inflation expectations under a flat Phillips curve.

The second strand of the literature focuses on firms' marginal cost and price markup as a potential cause of the missing deflation. Christiano et al. (2015) develop a model in which financial frictions raise firms' capital cost to hinge their marginal cost from falling in recessions. Kara and Pirzada (2016) introduce intermediate good prices in the Smets and Wouters (2007) model to take into account the rises of commodity prices in the data. On the other hand, Gilchrist et al. (2017), extending the model of consumer capital by Ravn et al. (2006), suggest that the liquidity needs during the financial crisis drove firms to raise their price markup given their nominal marginal cost at the expense of the future customer base.

Regarding the excessive disinflation, one of the earliest studies to point out the puzzle is Constâncio (2015). He refers to several hypotheses to settle the puzzle including anchoring of the inflation expectations and increased international competition, though formal analysis has not yet been provided in the literature, to my knowledge. On the empirical side, Albuquerque and Baumann (2017) propose to use a short-run labor market slack measure when estimating the Phillips curve, while Bobeica and Jarocinski (2017) emphasize the importance of the distinction between global and domestic factors to determine inflation.

This paper is distinct from these studies in a number of important dimensions. First of all, compared to the studies focusing on the inflation expectations, I show that my model can address both of the missing deflation and the excessive disinflation coherently. In contrast, if one argues that the missing deflation is driven by the rises of the inflation expectations relative to the rational expectations after the Great Recession, it is necessary to seek for another factor that prevents the recovery of inflation to address the subsequent excessive disinflation. Among the studies that investigate marginal cost and price markup, on the other hand, I focus on the wage channel to determine marginal cost.[3] In addition to the fact that the presence of DNWR is widely supported by micro evidence, I find the quantitative importance of it, that is, taking into account DNWR together with the ZLB explains most of the missing deflation that appears in a stylized New Keynesian model. On the other hand, Christiano et al. (2015), though they emphasize the role of capital cost, find that a negative productivity shock that offsets the decline of inflation as well as a negative demand shock is necessary to resolve the missing deflation. Similarly, the model of Kara and Pirzada (2016) that incorporates intermediate prices requires sufficiently high parameter values for price and wage rigidities to explain the puzzle.

Another class of literature that this paper is deeply related to is that of DNWR. Here I assess the studies that explore the aggregate implications of DNWR, while numerous studies have investigated micro evidence of it.[4] A seminal work by Akerlof et al. (1996) demonstrates that the long-run wage Phillips curve is no longer vertical in the presence of DNWR. In other words, involuntary unemployment does not decay even in the long-run, since once the actual wage exceeds the desired one the discrepancy between them is not eliminated without inflation. Benigno and Ricci (2011) analytically characterize the equilibrium with DNWR under a modern setting with optimizing agents. Applying their insights, recent studies argue that the upward sloping long-run wage Phillips curve due to DNWR together with the ZLB is a cause behind the jobless recoveries after the Great Recession (Schmitt-Grohé and Uribe (2017)) and secular stagnation (Eggertsson et al. (2017), Kocherlakota (2017)). Elsby (2009), on the other hand, claims that DNWR does not have significant effects on the aggregate wage growth since forward looking agents compress their wage hikes for a precautionary motive. Daly and Hobijn (2014), focusing on transition dynamics, show that DNWR bends the short-run wage Phillips curve as well, which accounts for the non-linear fluctuations of the unemployment rate. Schmitt-Grohé and Uribe (2016) develop a small open economy model to point out that DNWR is the fundamental cause of the high unemployment rate in the euro area in recent years.

A crucial difference of my model from the existing literature on DNWR is that I introduce DNWR into the New Keynesian setting with nominal price rigidity. It is worth pointing out that the studies on DNWR discussed above are conducted under flexible prices. More specifically, these models are developed either in the steady state with a constant inflation rate (Akerlof et al. (1996),

[3]It should be noted that existing literature has a tension between two distinct views on the marginal cost fluctuations after the Great Recession: Christiano et al. (2015) and Kara and Pirzada (2016) identify a dampened decline in marginal cost that prevents inflation from declining through a standard NKPC, whereas Gilchrist et al. (2017) claim that price markup increased upon the financial crisis, which implies marginal cost, defined as the inverse of price markup, declined a lot. While there are substantial debates on the cyclicality of marginal cost (price markup) in the literature (Basu (1995), Rotemberg and Woodford (1999), Nekarda and Ramey (2013), Bils et al. (2014), for example), in terms of its relation to inflation dynamics, my estimation results of the NKPC suggest that the marginal cost representation of the NKPC does not display significant changes after the Great Recession. Therefore, I take the standard NKPC in terms of marginal cost as given and study the consequence of DNWR through the formation of marginal cost.

[4]Discussion on the micro evidence of DNWR and its connection to this paper is provided in section 7.

Elsby (2009), Benigno and Ricci (2011), Schmitt-Grohé and Uribe (2017), Eggertsson et al. (2017), Kocherlakota (2017)) or under a time-varying but fully flexible inflation rate (Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016)). However, nominal price rigidity is an essential ingredient of the model for my purpose: studying inflation dynamics. In particular, nominal price rigidity is indispensable to capture the Phillips curve observed in data. I also find that the interaction between nominal price rigidity and DNWR leads to substantial persistence of real wage, which replicates sluggish movements of price variables and sizable fluctuations of real quantities in the data.

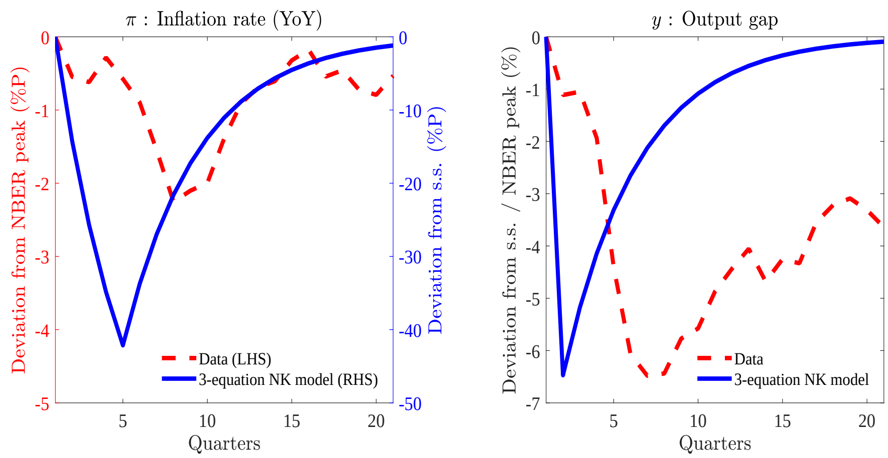
To this regard, several papers such as Kim and Ruge-Murcia (2009), Fahr and Smets (2010), and Aruoba et al. (2017) propose to employ a smooth asymmetric wage adjustment cost function to approximate DNWR. However, I find that the explicit DNWR constraint in my model endogenously generates a strong non-linear dynamics, and therefore the model is able to match the data under micro founded parameter values. Moreover, whereas all of Kim and Ruge-Murcia (2009), Fahr and Smets (2010), and Aruoba et al. (2017) use a perturbation method to derive an approximated solution around the steady state, my global solution method can capture the state-dependency of the model, which I find is essential to resolving the excessive disinflation.

On the methodological side, the model developed in this paper is classified as a heterogeneous agent model with aggregate uncertainty, which is initiated by Krusell and Smith (1998). This class of model has been used to study several dimensions of the economy in the existing literature, including asset and consumption dynamics (Krusell and Smith (1998), Krueger et al. (2016), etc.), search and matching (Krusell et al. (2010), Nakajima (2012), etc.), and price setting behavior (Nakamura and Steinsson (2010), Vavra (2013), etc.). Among others, my model is closely related to Gornemann et al. (2016), McKay and Reis (2016), and Blanco (2018), who apply the concept to the New Keynesian model, but distinct from them in that I examine the heterogeneity of individual wages arising from DNWR.

3 Motivating evidence

This section presents motivating evidence for the model analysis in the subsequent sections. First, using a stylized 3-equation New Keynesian model, I point out that a key assumption to bring about the difficulty in accounting for the inflation dynamics after the Great Recession is in the relationship between marginal cost and the output gap. Second, I provide empirical evidence to show that the assumption does not hold in the data. Specifically, I estimate two representations of the NKPC: the output gap representation and the marginal cost representation, to uncover that, whereas the farmer became flatter after the Great Recession, the latter remained stable. I assess the empirical relationship between marginal cost and the output gap in the data as well.

Figure 1: IR in the 3-equation New Keynesian model and data



Notes: The impulse responses (IR) of the 3-equation New Keynesian model are the responses to an exogenous innovation $e_{\epsilon,t}$. Size of shock is calibrated to match the drop of the output gap in data. For the series of data, the inflation rate is the year-on-year growth rate of the GDP implicit price deflator. The output gap is the one estimated by the Congressional Budget Office (CBO). Each data series is in the deviation from the business cycle peak before the Great Recession defined by the NBER (2007Q4).

3.1 Example in a 3-equation New Keynesian model

To see the core of the problem of a standard New Keynesian model in resolving the excessive disinflation and the missing inflation, I consider a stylized 3-equation linearized New Keynesian system that consists of the Euler equation (1), the NKPC (2), and the Taylor rule (3):

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}]) + \epsilon_t \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \tilde{\kappa} y_t \tag{2}$$

$$i_t = \delta_\pi \pi_t + \delta_y y_t \tag{3}$$

$$where \quad \epsilon_t = \rho_\epsilon \epsilon_{t-1} + e_{\epsilon,t}, \quad e_{\epsilon,t} \sim i.i.d.N(0, \sigma_\epsilon^2)$$

with $\tilde{\kappa} \equiv \kappa(\sigma + \eta)$ representing the slope of the NKPC. y_t, π_t, and i_t denote the output gap, the inflation rate, and the nominal interest rate. Each variable is in the log-deviation from the zero-inflation steady state. For simplicity, I suppose that the error term in the Euler equation ϵ_t, which follows an AR(1) process, is the only exogenous component of the economy. The derivation of the equations follows Walsh (2010). For the numerical exercise below, parameter values are set as follows: the discount factor $\beta = 0.995$, the relative risk aversion $\sigma = 2.00$, the inverse of the Frisch labor supply elasticity $\eta = 0.25$, the Taylor rule coefficients $\delta_\pi = 1.50, \delta_y = 0.25$, the slope of the NKPC $\kappa = 0.20$, and the persistence of the exogenous shock $\rho_\epsilon = 0.80$. Although these parameter values are in line with the literature, the choice of the parameter values is discussed in Section 5.

Figure 1 compares the impulse responses of the 3-equation New Keynesian model and the data after the Great Recession. It is immediate to see that the 3-equation New Keynesian model predicts massive deflation (-42 percentage point), whereas the actual inflation rate declined moderately (-2.3 percentage point). It is also notable that the inflation rate in the model quickly reverts toward the steady state value, while the recovery of the actual inflation were fairly sluggish with a sizable gap from the pre-crisis rate even 20 quarters after the NBER peak.

Among others, one of the key features to bring about these undesirable implications in the 3-equation New Keynesian model is the relationship between the output gap and marginal cost. To be precise, real wage and the households' marginal rate of substitution between consumption and hours worked should be equalized in a frictionless labor market. Real wage is proportional to the firms' marginal cost of producing one unit of output, whereas the marginal rate of substitution is related to households' consumption and hours worked, and therefore tied with the output gap in general equilibrium. In other words, the labor market equilibrium condition determines the relationship between the output gap and marginal cost. Indeed, a linear relationship between them can be derived in the stylized New Keynesian setting. Since inflation is pined down by the current and future marginal costs through the NKPC, it is quite difficult to obtain a small decline of inflation and a large drop of the output gap simultaneously as long as the linear relationship between the output gap and marginal cost is taken as given.

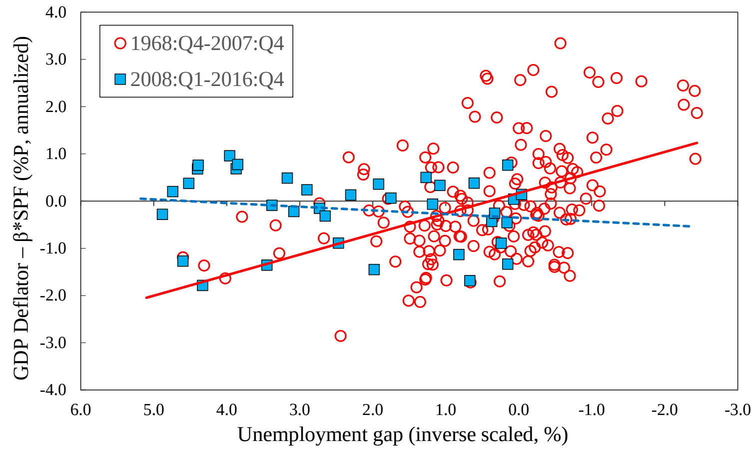
3.2 Empirical evidence

3.2.1 Estimation of the NKPC

This subsection estimates the NKPC in U.S. data. The NKPC is originally formulated from a firms' profit maximization problem under nominal price rigidity and therefore it relates firms' marginal cost to inflation, while the output gap representation is obtained as a consequence of general equilibrium. On the empirical side, early studies by Gali and Gertler (1999) and Sbordone (2002) obtain significant estimates for the slope parameter of the NKPC when using a measure of marginal cost rather than the output gap as a regressor. Both studies conclude that employing marginal cost is preferable for the estimation of the NKPC because it purely represents the firms' optimization behavior without imposing additional assumptions on the other parts of the economy. On the other hand, many of recent studies including Ball and Mazumder (2011), Murphy (2014), and Coibion and Gorodnichenko (2015) report the flattening of the NKPC after the Great Recession by estimating the output gap representation. Therefore, it would be worthwhile to examine whether the flattening is present in the marginal cost representation of the NKPC as well.

Before proceeding to a regression analysis, I plot the output gap representation of the NKPC in U.S. data in Figure 2. The fitted lines in the figure suggest that the output gap representation of the NKPC became flatter in the sample after 2008Q1 in line with findings of existing studies. It implies not only that the initial decline of inflation during the Great Recession was small but also that the recovery of inflation has been slow compared to the improvements of real economic activities.

Figure 2: The output gap representation of the NKPC in U.S. data



Notes: The x-axis is the unemployment gap as a proxy for the output gap (x_t). The y-axis represents the inflation rate minus the inflation expectation term ($\pi_t - \beta\pi_t^e$) where the discount factor β is calibrated to be 0.995. The inflation measure is the GDP implicit price deflator. The inflation expectation is the median forecast of the SPF. The slope of the fitted lines represents the slope of the NKPC in each sub-sample.

I estimate the following specification of the NKPC:

$$\pi_t = \beta\pi_t^e + \gamma\pi_{t-1} + \kappa x_t + e_t \tag{4}$$

Three modifications are added to the NKPC of Equation (2). First, the expectation term $\mathbb{E}_t[\pi_{t+1}]$ is replaced with the survey based expectation π_t^e. It reflects empirical finding by Adam and Padula (2011), Coibion and Gorodnichenko (2015), and Furhrer (2017) that using survey based expectation measures substantially improve the fit of a regression model. Second, following a convention of the literature such as Gali and Gertler (1999), the lagged inflation term π_{t-1} is included. Lastly, it should be noted that the error term e_t captures exogenous innovations to the current inflation such as markup shocks.

I use the GDP implicit price deflator as a measure of inflation. It is considered to be preferable to the Consumer Price Index (CPI) or the Personal Consumption Expenditures (PCE) inflation for the purpose of this regression analysis, because the inflation rate that appears in the NKPC should reflect the pricing behavior of domestic firms rather than the price index that domestic households face with. The measure of the inflation expectations is the median forecast of the GDP deflator in the Survey of Professional Forecast (SPF) provided by the Federal Reserve Bank of Philadelphia. The SPF is the only survey that asks the forecast of the GDP deflator. Two cases are considered for the choice of varialbe x_t: the output gap and marginal cost. Following Coibion and Gorodnichenko (2015), I use the unemployment gap as a proxy variable for the output gap.[5]

[5] The long-term unemployment gap provided by the CBO is used for a baseline estimation. However, the estimation result is quite similar when the short-term unemployment rate and the original series of the unemployment rate are used. Main results are robust for different proxy variables for other slack measures such as the detrended output and the output gap estimated by the CBO.

Regarding a series of marginal cost, since it is infeasible to directly observe firms' marginal cost, I follow the insights of Hall (1986) to resort to firms' optimizing behavior to estimate it. Specifically, I consider a cost minimization problem:

$$\min_{H_t, K_t} : \; P_t^H H_t + P_t^K K_t \tag{5}$$

$$s.t. \quad Y_t = F(H_t, K_t, Z_t) \tag{6}$$

where Y_t denotes output, Z_t technology, H_t labor inputs, K_t capital inputs. Production technology is given by F. Firms are assumed to be price taker in factor markets. The first order condition (FOC) for the problem takes the form:

$$P_t^J = \lambda_t \frac{\partial F_t}{\partial J_t} \quad for \; J = K, H \tag{7}$$

where λ_t denotes the Lagrangian multiplier that represents the nominal marginal cost of producing one unit of output. The FOC can be rearranged to:

$$MC_t \equiv \frac{\lambda_t}{P_t} = \underbrace{\left(\frac{\partial log(F_t)}{\partial log(J_t)}\right)^{-1}}_{(\theta_t^J)^{-1}} \underbrace{\frac{P_t^J J_t}{P_t Y_t}}_{s_t^J} \tag{8}$$

with θ_t^J and s_t^J being the output elasticity with respect to input J and the expenditure share of input J, respectively. Equation (8) allows one to construct the real marginal cost MC_t with observable variables s_t^J and θ_t^J. I use the labor share of income for the non-farm business sector to represent the expenditure share s_t^J. To estimate the elasticity θ_t^J, on the other hand, I impose an assumption on the functional form of the production technology F. Following Basu (1996), Gagnon and Khan (2005), and Nekarda and Ramey (2013), I assume the Cobb-Douglas production function with overhead labor (CDOH) for a baseline case. Alternative specifications of the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES) are investigated as well. Details are provided in Appendix A.

Table 1 presents the estimation result of the NKPC (4). Each coefficient without interaction is significant with intended sign. Regarding the changes of the slope parameter, in column 1 and 2 the coefficient of the output gap shrinks to less than a half once the observations after 2008Q1 are included. Moreover, the interaction term with the dummy variable for the post-Great Recession period is significantly negative in column 3. In contrast, the coefficient of marginal cost in column 4 and 5 is quite stable after the Great Recession with one standard error of each coefficient covering the other. The interaction term with the post-Great Recession dummy in column 6 is insignificant.

For robustness check, I assess the following alternative cases: (1) alternative specifications of marginal cost, (2) the purely forward looking NKPC, (3) the rational expectation assumption, (4) rolling regression, and (5) Markov-switching model of the parameter values. I also conduct (6) dynamic panel estimation using industry level data. Though details are presented in Appendix

Table 1: OLS estimation of the NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x_t	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.609***	0.570***	0.621***	0.420***	0.468***	0.441***
	(0.082)	(0.078)	(0.080)	(0.080)	(0.074)	(0.078)
π_{t-1}	0.423***	0.448***	0.410***	0.573***	0.526***	0.552***
	(0.077)	(0.075)	(0.076)	(0.081)	(0.075)	(0.080)
x_t	0.332***	0.158***	0.310***	0.236***	0.186***	0.212***
	(0.059)	(0.046)	(0.056)	(0.077)	(0.062)	(0.073)
$postGR_t \times \pi_t^e$			0.054			0.372
			(0.218)			(0.223)
$postGR_t \times \pi_{t-1}$			-0.164			-0.362
			(0.221)			(0.234)
$postGR_t \times x_t$			-0.333***			-0.0796
			(0.091)			(0.130)
Adjusted R^2	0.956	0.948	0.950	0.953	0.947	0.947
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The sign of the coefficient of the unemployment gap is flipped for a comparison purpose. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to when the SPF became available.

A, these alternative cases confirm the baseline result in that the coefficient of marginal cost is stable over time whereas the coefficient of the output gap drops after the Great Recession or is insignificant even before the Great Recession in some specifications.[6]

3.2.2 Relationship between marginal cost and the output gap

This subsection examine empirical relationship between marginal cost and the output gap using industry level data of KLEMS 2017. An advantage of employing industry level data is that the intermediate share is available for measuring marginal cost. To this regard, a number of studies (Basu (1995), Nekarda and Ramey (2013), and Bils et al. (2014), for example) suggest that intermediate inputs have desirable features in many dimensions. For instance, adjustment costs for intermediates are considered to be low relative to those for capital or labor. In addition, the assumption of no overhead component seems more defensible for intermediates. Using industry

[6]For example, the coefficient of the output gap is found to be insignificant under the rational expectation assumption regardless of sample periods. The result is consistent with the existing literature. Adam and Padula (2011) find that the coefficient of the output gap is significant only when a survey based expectation measure is used instead of the rational expectation assumption.

Table 2: Dynamic panel estimation of the cyclicality of marginal cost to the output gap measures

	Dependent variable : $IntShare_{i,t}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Linear	post-GR	LQT	HQT	Linear	post-GR	LQT	HQT
$Output_{i,t}$	0.316^{+++}	0.371^{+++}	0.395^{+++}	0.258^{+++}				
	(0.031)	(0.035)	(0.048)	(0.043)				
$postGR_t \times Output_{i,t}$		-0.245^{***}						
		(0.076)						
$LQT_{i,t} \times Output_{i,t}$			-0.164^{**}					
			(0.078)					
$HQT_{i,t} \times Output_{i,t}$				0.129^{*}				
				(0.066)				
$UnempGap_t$					0.676^{+++}	1.159^{***}	1.118^{***}	0.242
					(0.167)	(0.362)	(0.410)	(0.223)
$postGR_t \times UnempGap_t$						-1.004^{**}		
						(0.393)		
$LQT_t \times UnempGap_t$							-0.902^{**}	
							(0.438)	
$HQT_t \times UnempGap_t$								1.074
								(1.064)
Industry FE	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	NO	NO	NO	NO
N of ind.	60	60	60	60	60	60	60	60
N of total obs.	1,800	1,800	1,800	1,800	1,800	1,800	1,800	1,800

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: Standard errors are reported in parentheses. The intermediate share and the detrended output are taken log and detrended by the Hamilton filter. The sign of the coefficient of the unemployment gap is flipped for a comparison purpose. $postGR_t$ is a dummy variable that takes 1 after year 2008. $LQT_{i,t}$ and $HQT_{i,t}$ is a dummy variable that takes 1 if the observation is in the lower and higher 25 percentile of the sample, respectively. Sample is annual data from 1985 to 2014 for 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing. In column 5-8, dummy variables for year 2008 and 2009 are included to control for the large variations during the financial crisis.

level data removes composition bias among industries as well.[7]

I run the dynamic panel regression of marginal cost on the output gap. Measures of the output gap are the industry level detrended output and the aggregate unemployment gap. Table 2 presents that the coefficient of the detrended output is significantly positive in column 1-4. The finding supports the procyclicality of marginal cost (counter-cyclical markup) in line with previous studies such as Basu (1995), Gali et al. (2007), and Bils et al. (2014). More importantly, the data identifies convexity in the relation between marginal cost and the detrended output. To

[7]Despite these desirable properties, the survey based inflation expectations are not available for each industry. Therefore, for the estimation analysis of the NKPC in the previous subsection, I use the labor share in the aggregate data in the baseline estimation. However, the estimation result of the NKPC using the intermediate share as a measure of marginal cost confirm the baseline results. Details are provided in Appendix A.

be precise, the interaction term with the post-Great Recession dummy is significantly negative in column 2, implying that marginal cost did not decline as much as the detrended output did after the Great Recession. Interestingly, the interaction term with the lower quartile in column 3 is negative, whereas that with the higher quartile in column 4 is positive. The result suggests that the convex relationship is present over business cycles in general, though the convexity is particularly significant after the Great Recession. These features are supported with respect to the aggregate unemployment gap in column 5-8 as well.

In sum, the empirical evidence in this section suggests that the marginal cost representation of the NKPC remained stable after the Great Recession, while I confirm the flattening of the output gap representation in line with the existing literature. I also find that marginal cost has a convex relationship with the output gap, and the convexity significantly appeared after the Great Recession. The evidence casts a doubt on one of the key features in a stylized New Keynesian model that the output gap and marginal cost has a linear relationship. In the next section, therefore, I develop a model in which labor market friction arising from DNWR creates a wedge between them. The model formulates non-linearity in the observed Phillips curve relationship between inflation and the output gap, while keeping the marginal cost representation of the NKPC unchanged.

4 Model

This section develops a DSGE model that embeds the DNWR constraint for individual workers. Other parts of the economy share many features of a standard New Keynesian model in the literature such as the one by Erceg et al. (2000), Ireland (2004), and Christiano et al. (2005). The economy consists of monopolistically competitive firms that set their prices with the quadratic adjustment cost *á la* Rotemberg (1982), households who make saving-consumption decision and supply differentiated labor service to the production sector, and the central bank that follows the Taylor (1993) type nominal interest rate policy to stabilize inflation and the output gap.

4.1 Households

In the economy, there is a continuum of households indexed by j on the unit interval, each of whom supplies a differentiated labor service to the production sector. The aggregate labor supply has the Dixit-Stiglitz form:

$$H_t \equiv \left(\int_0^1 h_t(j)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}} \tag{9}$$

where θ_w represents the labor demand elasticity. The user of labor service minimizes the cost of using a given amount of composite labor inputs, taking each labor service's wage as given. The FOC for the cost minimization problem leads to the individual labor demand function:

$$h_t(j) = \left(\frac{w_t(j)}{W_t} \right)^{-\theta_w} H_t \tag{10}$$

where the aggregate wage index W_t is defined as

$$W_t \equiv \left(\int_0^1 w_t(j)^{1-\theta_w} dj \right)^{\frac{1}{1-\theta_w}} \tag{11}$$

The utility function of each household j is assumed to be additive separable in CRRA utility from consumption $c_t(j)$ and CRRA disutility from labor $h_t(j)$ with parameter σ and η respectively. The disutility from labor is subject to an uninsurable idiosyncratic shock $\chi_t(j)$, which follows an *i.i.d.* normal distribution. The time-varying discount factor β_t is exogenous and common for each household. It captures exogenous changes of households' preference. The expected lifetime utility is given by

$$\begin{aligned} &\mathbb{E}_t \left[\sum_{s=0}^{\infty} D_{t,t+s} \left(\frac{1}{1-\sigma} c_t(j)^{1-\sigma} - \frac{1}{1+\eta} e^{\chi_t(j)} h_t(j)^{1+\eta} \right) \right] \\ &where \quad \chi_t(j) \sim i.i.d.N(0, \sigma_\chi^2) \\ &D_{t,t+s} = \beta_{t+s} D_{t,t+s-1} \end{aligned} \tag{12}$$

The aggregate discount factor β_t follows an AR(1) process:

$$log(\beta_t) = (1-\rho_d) log(\bar{\beta}) + \rho_d log(\beta_{t-1}) + \epsilon_{d,t} \ , \ \epsilon_{d,t} \sim i.i.d.N(0, \sigma_d^2) \tag{13}$$

where $\bar{\beta}$ represents the unconditional mean of β_t. One can interpret a positive value of d_t as a contractionary discount factor shock where households lose their desire to consume in the current period.

Household j's budget constraint in period t is given by

$$c_t(j) + \frac{a_t(j)}{P_t} \leq (1+\tau_w) \frac{w_t(j)}{P_t} h_t(j) + R_{t-1} \frac{a_{t-1}(j)}{P_t} + \frac{T_t(j)}{P_t} + \frac{\Phi_t(j)}{P_t} \tag{14}$$

where $a_t(j)$ is the amount of asset holding, $T_t(j)$ is the lump sum transfer, and $\Phi_t(j)$ is the share of producer's profits distributed to household j. P_t, R_{t-1}, and τ_w denote the aggregate price index, the gross nominal interest rate, and the labor subsidy, respectively.

Household's nominal wage might be subject to the DNWR constraint. I assume that $1-\alpha$ fraction of households is not allowed to reduce their nominal wages in each period with $0 < \alpha < 1$, whereas the remaining α fraction of them is free to change their wages without the constraint:

$$w_t(j) \geq w_{t-1}(j) \quad with\ prob.\ 1-\alpha \tag{15}$$

This assumption reflects a well known empirical fact that nominal wage reduction is rare to occur. For instance, Baratteiri et al. (2014), who study the frequency of individual nominal wage changes in the Survey of Income and Program Participation (SIPP) during 1996-1999, report that nominal wage reduction only corresponds to 12.3 percent of all the non-zero nominal wage changes after correcting for measurement errors.

Each household j maximizes the expected lifetime utility (12) by choosing her consumption $c_t(j)$, asset holding $a_t(j)$, and nominal wage $w_t(j)$ subject to her budget constraint (14), individual labor demand (10), and the DNWR constraint (15) if she is subject to it. The FOCs for the problem take the form:

$$\mathbb{E}_t\left[\beta_t\left(\frac{c_{t+1}(j)}{c_t(j)}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right]=1 \tag{16}$$

$$\psi_t(j)=\left(\frac{w_t(j)}{P_t}-\frac{\bar{\mu}_w}{1+\tau_w}mrs_t(j)\right)\left(\frac{1+\tau_w}{\bar{\mu}_w}c_t(j)^{-\sigma}\frac{\theta_w h_t(j)}{w_t(j)}\right)+\beta_t\mathbb{E}_t[\psi_{t+1}(j)] \tag{17}$$

$$where \quad mrs_t(j)\equiv\frac{e^{\chi_t(j)}h_t(j)^{\eta}}{c_t(j)^{-\sigma}}$$

with $\bar{\mu}_w\equiv\theta_w/(\theta_w-1)$ and $\Pi_t\equiv P_t/P_{t-1}$. $\bar{\mu}_w$ is the steady state wage markup stemming from the monopolistic power of each household for her differentiated labor service, and $mrs_t(j)$ is the marginal rate of substitution of household j. $\psi_t(j)$ denotes the Lagrange multiplier for the DNWR constraint, which represents the shadow value of easing the DNWR constraint by one unit. The complementary slackness conditions for the DNWR constraint are given by

$$\psi_t(j)\geq 0 \tag{18}$$

$$\psi_t(j)(w_t(j)-w_{t-1}(j))=0 \tag{19}$$

In the following analysis, I impose two additional assumptions to focus on my main points keeping the model tractable. First, I assume that each household has an access to a complete insurance market for consumption, though she is still subject to an uninsurable idiosyncratic labor disutility shock. Then, consumption is identical across households:

$$c_t(j)=C_t \tag{20}$$

Second, following Erceg et al. (2000), I assume that the labor subsidy is set to remove the steady state wage markup:

$$1+\tau_w=\bar{\mu}_w \tag{21}$$

Due to these assumptions, the FOCs (16) and (17) can be simplified to:

$$\mathbb{E}_t\left[\beta_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right]=1 \tag{22}$$

$$\psi_t(j)=\left(\frac{w_t(j)}{P_t}-mrs_t(j)\right)\left(C_t^{-\sigma}\frac{\theta_w h_t(j)}{w_t(j)}\right)+\beta_t\mathbb{E}_t[\psi_{t+1}(j)] \tag{23}$$

The optimality conditions for individual wage setting are characterized by (18), (19), and (23). If it were not for the DNWR constraint, then, $\psi_t(j)=0$ would hold for all j and t. In that case, the optimality conditions are reduced to

$$\frac{w_t^f(j)}{P_t^f}=mrs_t^f(j) \tag{24}$$

where x_t^f denotes the variable x_t under flexible prices and wages. Equation (24) formulates an optimality condition under flexible wages to equalize the real wage to the marginal rate of substitution. On the other hand, in the presence of the DNWR constraint, the complementary slackness conditions for the constraint imply that either of the following has to hold true:

$$w_t(j) = w_{t-1}(j) \tag{25}$$

or

$$\psi_t(j) = 0 \tag{26}$$

The former condition corresponds to the case where the DNWR constraint binds in the current period, whereas the latter the case where it does not. The latter condition is rearranged by using (23):

$$\frac{w_t(j)}{P_t} = mrs_t(j) - \beta_t \mathbb{E}_t[\psi_{t+1}(j)] \left(C_t^{-\sigma} \frac{\theta_w h_t(j)}{w_t(j)} \right)^{-1} \tag{27}$$

The first term in the RHS of (27) coincides with the optimal wage under flexible wages, whereas the second term represents the reserved wage hike due to the likelihood of the DNWR constraint binding in the future periods. It is worth pointing that the optimal wages implied by (27) are weakly lower than their marginal rate of substitution since $\psi_t(j)$ is non-negative by (18). In other words, DNWR endogenously generates upward wage stickiness. Intuitively, a household internalizes the possibilities that her DNWR constraint might bind in future periods, and therefore she desires to hold some buffer to prevent the future constraint from binding even if the constraint does not bind in the current period. This property is in line with the finding of the previous studies such as Elsby (2009), Benigno and Ricci (2011), and Daly and Hobijn (2014). To summarize the conditions above, the optimal wage set by household j who are subject to the DNWR constraint follow the rule:

$$w_t(j) = max\left\{ w_t^d(j), w_{t-1}(j) \right\} \tag{28}$$

where the desired wage $w_t^d(j)$ satisfies the condition (27).

4.2 Firms

There is a continuum of monopolistically competitive firms indexed by i on the unit interval, each of which produces a differentiated good. The production technology available for the firm producing good i is given by

$$y_t(i) = Z_t h_t(i) \tag{29}$$

$$where \quad h_t(i) = \left(\int_0^1 h_t(i,j)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}} \tag{30}$$

Technology Z_t is exogenous and common for each firm. Firm i uses the composite labor inputs $h_t(i)$, where $h(i,j)$ denotes the labor service supplied by household j and used in firm i.

The cost minimization problem to determine the labor inputs $h_t(i)$ is given by

$$\min_{h_t(i)} : \frac{W_t}{P_t} h_t(i) \quad s.t. \quad (29) \tag{31}$$

The FOC takes the form:

$$mc_t(i) = \frac{W_t}{Z_t P_t} \equiv MC_t \tag{32}$$

where MC_t is the real marginal cost of producing one unit of output. (32) implies that the marginal cost is identical across firms. It is because each firm has the identical labor demand elasticity and takes labor service's wages as given.

I next formulate the firms' profit maximization problem. Firms face with the quadratic price adjustment cost formulated of Rotemberg (1982) with parameter ϕ governing the degree of price stickiness. A firm chooses its price to maximize the expected profit subject to the individual good demand function that is analogous to the individual labor demand function:

$$\max_{p_t(i)} : \mathbb{E}_t \left[\sum_{s=0}^{\infty} D_{t,t+s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma} \Phi_{t+s}(i) \right] \tag{33}$$

$$where \quad \Phi_t(i) = (1+\tau_p) \frac{p_t(i)}{P_t} y_t(i) - MC_t y_t(i) - \frac{\phi}{2} \left(\Pi_t(i) - \Pi^* \right)^2 C_t$$

$$s.t. \quad y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t \tag{34}$$

τ_p denotes the production subsidy. As in the households' problem, the production subsidy is set to cancel out the steady state price markup arising from firms' monopolistic power:

$$1 + \tau_p = \bar{\mu}_p \tag{35}$$

where $\bar{\mu}_p \equiv \theta_p/(\theta_p - 1)$. In the symmetric equilibrium, each firm sets identical price and the FOC for the profit maximization problem yields the NKPC:

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \kappa \left(MC_t - 1 \right) \tag{36}$$

where MC_t is defined in (32). $\kappa \equiv \theta_p/\phi$ represents the slope of the NKPC. The aggregate production function in the symmetric equilibrium is given as follows.

$$Y_t = Z_t H_t \tag{37}$$

4.3 Central bank and government

In the baseline model, I assume a Taylor (1993) type monetary policy rule where the central bank sets the gross nominal interest rate R_t to stabilize the gross inflation rate Π_t around its target level Π^* and the output gap Y_t/Y_t^f: [8]

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\delta_y} \tag{38}$$

where $R^* = \Pi^*/\beta$ and Y_t^f denotes the output under flexible prices and wages.

In this economy, the government is passive in the sense that it levies lump sum tax on households and distributes it as production and labor subsidies to firms and households with a balanced budget:

$$\int_0^1 T_t(j)dj = \tau_w \int_0^1 w_t(j)h_t(j)dj + \tau_p \int_0^1 p_t(i)y_t(i)di \tag{39}$$

Notice that the government does not affect households' consumption and saving decision.

4.4 Market clearing

Market clearing conditions are given as follows.

$$A_t \equiv \int_0^1 a_t(j)dj = 0 \tag{40}$$

$$Y_t = C_t + \frac{\phi}{2}(\Pi_t - \Pi^*)^2 C_t \tag{41}$$

$$H_t = \int_0^1 h_t(i)di \tag{42}$$

The asset market clearing (40) is trivial because consumption is shared across households and therefore it is not necessary to keep track of individual asset holdings to characterize equilibrium. The goods market clearing (41) should hold in the aggregate level since I restrict the attention to the symmetric equilibrium. It should be noticed that the labor market clearing (42) is automatically satisfied in the symmetric equilibrium as well.

4.5 Equilibrium

Definition. *A recursive competitive equilibrium is a household's policy function for individual real wages* $\tilde{w} = h(\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z)$, *a policy function for a set of aggregate jump variables* $X \equiv \{C, Y, H, \Pi, R\} = f(g_{-1}, \beta, Z)$, *and a law of motion* Γ *for cross-sectional density of individual real wages* g, *such that*

(i) a household's policy function h *solves a recursive wage setting problem,*

[8] I introduce the ZLB to conduct a counterfactual analysis for the Great Recession in Section 8.

$$
\begin{aligned}
V^{dnwr}\Big(\tilde{w}_{-1},\chi;g_{-1},\beta,Z\Big) &= \max_{\tilde{w}}: \; -\frac{1}{1+\eta}e^{\chi}h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\big(\tilde{w},\chi';g,\beta',Z'\big)|g_{-1},\beta,Z\Big] \\
&\quad s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H \\
&\quad\quad\quad\;\; \tilde{w} \geq \tilde{w}_{-1}/\Pi \\
V^{no}\Big(\chi;g_{-1},\beta,Z\Big) &= \max_{\tilde{w}}: \; -\frac{1}{1+\eta}e^{\chi}h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\big(\tilde{w},\chi';g,\beta',Z'\big)|g_{-1},\beta,Z\Big] \\
&\quad s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H \\
where \quad \mathbb{E}\Big[V\big(\tilde{w},\chi';g,\beta',Z'\big)|g_{-1},\beta,Z\Big] &= (1-\alpha)\mathbb{E}\Big[V^{dnwr}\big(\tilde{w},\chi';g,\beta',Z'\big)|g_{-1},\beta,Z\Big] \\
&\quad + \alpha\mathbb{E}\Big[V^{no}\big(\chi';g,\beta',Z'\big)|g_{-1},\beta,Z\Big]
\end{aligned}
$$

with $\tilde{W}$ being the aggregate real wage generated by the cross-sectional density g and C, H, and Π being consistent with the aggregate policy function f.
(ii) an aggregate policy function f solve the Euler equation (22), the NKPC (36), the monetary policy rule (38), the production function (37), and the market clearing conditions (41),
(iii) a law of motion Γ is generated by g, that is, the cross-sectional density g satisfies a recursive rule:

$$
g = \Gamma(g_{-1},\beta,Z).
$$

4.6 Analytical example

I briefly discuss the key mechanism of the model before proceeding to a numerical solution to it. For the example below, I impose several additional assumptions to analytically characterize equilibrium. To be precise, I consider log-utility from consumption and linear-disutility from labor, i.e. $\sigma = 1$ and $\eta = 0$. I also assume that there are no idiosyncratic disutility shocks.

Under flexible prices and wages, the labor market equilibrium condition requires the marginal product of labor MPL and the marginal rate of substitution between consumption and hours worked MRS should be equalized with each other through real wage:

$$
\begin{aligned}
MPL_t^f = \frac{W_t^f}{P_t^f} &= MRS_t^f \qquad (43) \\
where \quad MPL_t^f &= Z_t \\
MRS_t^f &= Y_t^f
\end{aligned}
$$

Note that j notation for each household is dropped in (43) since the marginal rate of substitution does not have idiosyncratic components in this example. Wage and price markups do not appear in (43) because both markups are constant under flexible prices and wages and the steady state markups are canceled out with the labor and production subsidy. In the presence of DNWR and price stickiness, on the other hand, markups are no longer constant. The labor market equilibrium condition takes the form:

$$
\begin{aligned}
MC_t MPL_t &= \frac{W_t}{P_t} = \mu_{w,t} MRS_t \qquad (44)\\
where \quad MPL_t &= Z_t \\
MRS_t &= Y_t
\end{aligned}
$$

The aggregate wage markup $\mu_{w,t}$ summarizes the wedge between the real wage and the marginal rate of substitution due to the DNWR constraint. On the other hand, $MC_t (\equiv 1/\mu_{p,t})$ captures the fluctuations in the real marginal cost resulting from imperfect price adjustment due to the nominal price rigidity. By combining (43) and (44) and taking logarithm of both sides, the relationship between the output gap and marginal cost is given as follows.

$$
\hat{MC}_t = (\hat{Y}_t - \hat{Y}_t^f) + \hat{\mu}_{w,t} \qquad (45)
$$

where I define $\hat{x}_t \equiv log(x_t)$. The first term in the RHS of (45) is the definition of the output gap, whereas the second term is the wage markup arising from DNWR.

I now examine the effect of the wage markup due to DNWR on price inflation. It is immediate to see the wage markup appears in the output gap representation of the NKPC by substituting (45) into the linearized version of (36):

$$
\begin{aligned}
\pi_t &= \beta \mathbb{E}_t [\pi_{t+1}] + \kappa \hat{MC}_t \\
&= \beta \mathbb{E}_t [\pi_{t+1}] + \kappa (\hat{Y}_t - \hat{Y}_t^f) + \kappa \hat{\mu}_{w,t} \qquad (46)
\end{aligned}
$$

where $\pi_t \equiv log(\Pi_t)$. Equation (45) and (46) suggest that DNWR creates a time-varying wedge between the output gap and marginal cost and it works as a shift parameter of the output gap representation of the NKPC. In recessions when the wedge increases due to the binding DNWR constraint, the output representation of the NKPC shifts up, which makes the observed relationship between inflation and the output gap flatter. To quantify these implications, I solve the model numerically in the next section.

5 Numerical Method and Calibration

This section presents an equilibrium computation method to solve the model developed in the previous section and explain my calibration strategy.

5.1 Modified Krusell-Smith algorithm

I present an equilibrium computation method to solve the model numerically. To this end, a perturbation method, which is used to solve a wide class of DSGE models in the literature, cannot be applied to the model. The first reason is non-linearity. Since the occasionally binding DNWR constraint makes the individual policy function kinked, the function is no longer differentiable. The presence of the ZLB is another source of non-linearity. In addition, the DNWR constraint introduces heterogeneity of wages among households. This is due to the state-dependent nature of the DNWR constraint. Unlike a time-dependent constraint such as the staggered contract of Calvo (1983), whether the DNWR constraint binds or not crucially depends on the previous period's wages. Therefore, in order to characterize equilibrium, it is necessary to keep track of the history of individual wages, i.e. cross-sectional wage distribution.

For these reasons, I apply the Krusell-Smith algorithm to the model. Since cross-sectional distribution is an infinite dimensional object, it is in practice impossible to track all the information in it. To this regard, Krusell and Smith (1998) propose an approximated equilibrium where each agent perceives the evolution of aggregate state variables as being a function of a small number of moments of cross-sectional distribution. Adopting their insight, I assume that the aggregate endogenous state variable, real wage $\tilde{W}_t$, is governed by the following aggregate law of motion (ALM):

$$\tilde{W}_t = \Gamma(\tilde{W}_{t-1}, \beta_t, Z_t) \tag{47}$$

An important challenge is that, even though the original Krusell-Smith algorithm requires aggregate jump variables to have a closed form solution in terms of aggregate state variables for using the ALM to track the state of the economy, that condition is not satisfied in the New Keynesian setting of this paper. Therefore, I propose a modified algorithm. Specifically, given a guess for the ALM of the aggregate state variable (47), I first solve for the aggregate jump variables, consumption C_t, hours worked H_t, price inflation Π_t, and nominal interest rate R_t, as general equilibrium outcomes of the aggregate part of the economy. Notice that the aggregate part of the economy consists of the 3-equation New Keynesian system, the Euler equation (22), the NKPC (36), and the Taylor rule (38), as well as the production function (37) and the market clearing conditions (41), and it is independent of individual workers' behavior conditional on the aggregate real wage. Then, given all the aggregate variables, an individual variable, real wage of each worker $\tilde{w}_t(j)$, can be obtained as a solution to an individual wage setting problem. Finally, I can aggregate the individual variables to recover the aggregate state variable and update the initial guess for the ALM. To address non-linearity stemming from DNWR and the ZLB, I used a global method in each step. Details of the computation algorithm are provided in Appendix B.

5.2 Calibration

Due to the complexity of my model, I follow a calibration strategy to set parameter values. The time frequency is quarterly. The externally fixed parameters are listed in Panel (A) of Table 3. The choice of the discount factor β and the target inflation rate Π^* corresponds to the annual real interest rate of 2 percent and the annual price inflation rate of 2 percent, respectively. The relative risk aversion of households σ is set at 2.0 and the inverse of the Frisch labor supply elasticity η

is at 0.25, which is in line with the existing literature. The values of θ_w and θ_p imply the steady state markup is 12.5 percent. I follow Fernández-Villaverde et al. (2015) to set $\delta_\pi = 1.50$ and $\delta_y = 0.25$. The value of the degree of price stickiness ϕ is calibrated according to the frequency of individual price changes reported by Nakamura and Steinsson (2008). They find that the median frequency excluding temporary sales is 11-13 percent per month, which implies the slope of the NKPC is around 0.20 and the corresponding parameter value is $\phi = 45.0$ in my model. Notice that the parameter value implies that 1 (5) percentage point deviation of inflation from its trend generates 0.225 (5.625) percent loss of consumption.[9] Although previous studies in the New Keynesian literature tend to use higher values for the price stickiness parameter to reproduce the persistence of the actual inflation, I investigate whether the model can account for the data under the parameterization that is consistent with micro evidence.

The parameters regarding cross-sectional wage distribution are calibrated to match the empirical distribution in U.S. data. I choose parameter values to minimize the quadratic distance between the moments of the stationary distribution of individual wage changes in the model and the target moments in data by using a grid search method. The target moments and the calibrated parameter values are listed in Panel (B) of Table 3.

In the following, I focus on the consequence of exogenous variations of discount factor β_t, while keeping technology constant at the unity, i.e., $Z_t = \bar{Z} = 1$.[10] This is because a plenty of evidence in the literature suggests that a demand side shock, in particular, a wedge in the intertemporal substitution, is the key determinant of the severe contractions during the Great Recession. Many of previous studies reach that conclusion by a reduced from regression analysis (Hall (2011)) and an estimation analysis of a structural model (Justiniano et al. (2011), Christiano et al. (2014), Gust et al. (2017), etc.). To this regard, as is pointed out by Justiniano et al. (2011), the time-varying discount factor is a parsimonious way to represent the shock. For parameterization, the AR(1) coefficient of the discount factor ρ_d and the standard deviation of innovations to it σ_d are calibrated to match the persistence and the standard deviation of the real GDP in the post-war U.S. data. The calibrated parameters are listed in Panel (C) of Table 3.

[9]The consumption loss is calculated as follows. $\frac{\phi}{2}(\Pi_t - \Pi^*)^2 * 100 = 45/2 * 0.01^2 * 100 = 0.225(\%)$

[10]For a curious reader, the effect of a technology shock is investigated in Appendix C.

Table 3: Calibrated parameters

Panel (A): Fixed parameters

Description	Symbol	Value	Source
Average discount factor	$\bar{\beta}$	0.995	S.S. real interest rate = 2.0% (annualized)
Relative risk aversion	σ	2.00	IES = 0.5
Inverse of Frisch labor supply elasticity	η	0.25	King and Rebelo (1999)
Labor demand elasticity	θ_w	9.00	S.S. wage markup = 12.5%
Consumption demand elasticity	θ_p	9.00	S.S. price markup = 12.5%
Price adjustment cost	ϕ	45.0	Slope of NKPC = 0.20
(Corresponding Calvo parameter)	-	(0.64)	Nakamura and Steinsson (2008)
Coefficient of Π in the Taylor rule	δ_π	1.50	Fernández-Villaverde et al. (2015)
Coefficient of Y in the Taylor rule	δ_y	0.25	same as above
Target level of Π	Π^*	1.005	S.S. inflation rate = 2.0% (annualized)

Panel (B): Calibrated parameters for the cross-sectional wage distribution

Description	Symbol	Value	Source
Target moment:			
Fraction of workers with non-zero wage changes	-	0.266	Barattieri et al. (2014)
S.D. of individual wage changes (annual)	-	0.108	Fallick et al. (2016)
Calibrated parameter:			
Fraction of workers without being subject to the DNWR constraint	α	0.0610	Internally calibrated
S.D. of idiosyncratic labor disutility shock	σ_χ	0.1540	same as above

Notes: Barattieri et al. (2014) identify the fraction of workers with non-zero wage changes to be between 0.211 and 0.266 depending on the assumptions they use in their estimation. I use the most conservative value 0.266 in terms of generating wage stickiness, since the model exclude any other possibilities to generate wage rigidity than DNWR.

Panel (C): Calibrated parameters for exogenous processes

Description	Symbol	Value	Source
Target moment:			
First-order autocorr. of output	-	0.85	HP-filtered real GDP
S.D. of output	-	0.0155	same as above
Calibrated parameter:			
AR(1) coefficient of discount factor	ρ_d	0.865	Internally calibrated
S.D. of innovations of discount factor	σ_d	0.00562	same as above

Notes: Sample period for the real GDP is from 1955Q1 to 2007Q4. The end of the sample is determined to exclude the ZLB periods.

Figure 3: Stationary distribution of non-zero wage changes

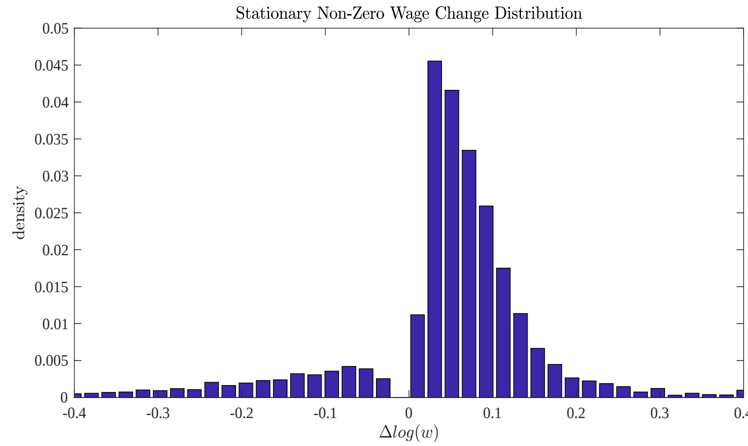


Table 4: Selected moments of stationary distribution

Description	Model	Data	Source
Targeted moment:			
Fraction of workers with non-zero wage changes	0.266	0.266	Barattieri et al. (2014)
S.D. of individual wage changes (annual)	0.108	0.108	Fallick et al. (2016)
Untargeted moment:			
Fraction of wage cuts out of non-zero wage changes	0.147	0.123	Barattieri et al. (2014)
		0.22, 0.32	Author's calculation based on Elsby et al. (2016)
Mean of wage changes (annual)	0.019	0.033	Fallick et al. (2016)
Median of wage changes (annual)	0.0098	0.028	Fallick et al. (2016)

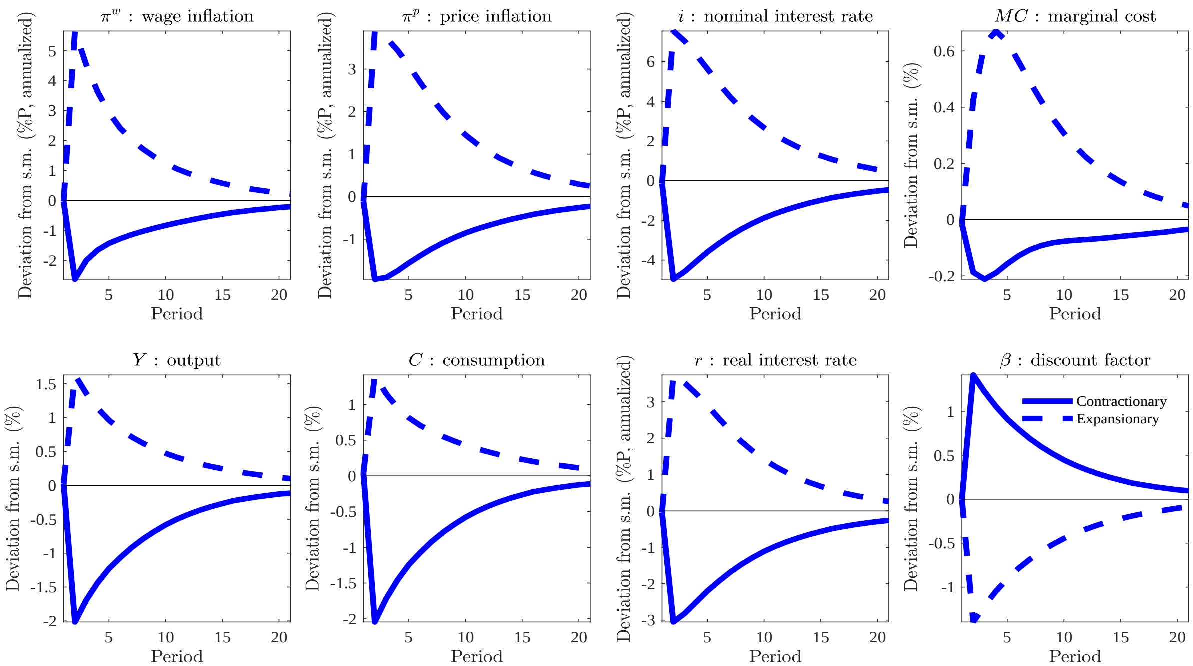
Notes: The data source and the sample period of each paper is as follows; Barattieri et al. (2014): SIPP, 1996-1999. Fallick et al. (2016): ECI, 1982-2014. Elsby et al. (2016): CPS, 1980-2012. For the fraction of wage cuts out of non-zero wage changes, 0.22 is for hourly paid workers and 0.32 is for non-hourly paid workers. Data frequency is quarterly, unless otherwise noted. The higher order moments of the distribution are not reported since I find that they are sometimes sensitive to small changes in parameter values. It might be because a large mass of workers is at the zero wage change. To this regard, Fallick et al. (2016) report the skewness of the distribution in the data takes positive and negative values in each year without a clear pattern.

6 Numerical Results

6.1 Stationary distribution

Figure 3 displays the stationary distribution of non-zero wage changes in the calibrated model. The definition of the stationary equilibrium is provided in Appendix B. Selected moments are reported in Table 4. The model replicates key features of the empirical distribution including: (1) a large spike at zero (not shown in the figure), (2) much less individuals with nominal wage reductions than increases, (3) a discrete difference in the density between the positive and negative sides around zero, and (4) higher mean than median. It should be noted that I do not target the asymmetry of the empirical distribution when calibrating parameters. Instead, these properties arise as a consequence of the specifications of the model.

Figure 4: GIR to a 2 S.D. discount factor shock



Notes: The x-axes represent the time horizon after the initial shock. The y-axes are in terms of the deviation from the stochastic mean (s.m.). The GIR are the conditional expectation on the initial shock. The construction of the GIR is provided in Appendix B.

6.2 Generalized impulse responses

The generalized impulse responses (GIR) to a 2 S.D. discount factor shock are presented in Figure 4. The construction of the GIR is provided in Appendix B. A discount factor shock, as a demand side shock, generates comovements among quantity and price variables. More importantly, the responses of wage growth, price inflation, and marginal cost display strong asymmetry to contractionary and expansionary shocks. They are much more sluggish downward than upward in the presence of DNWR. On the other hand, the responses of output and consumption are larger to a contractionary shock. It implies that quantity variables are adjusted instead of price variables as a consequence of general equilibrium. It should be noted that the asymmetry of quantity variables are relatively smaller than that of price variables, because the concavity of utility function makes households resist decline of consumption more strongly than they appreciate increase of it, and that effect partly offsets the asymmetry arising from DNWR. Moreover, the half-lives of the price variables are longer upon a contractionary shock. For example, the half-lives of price inflation are 10 quarters for a contractionary shock whereas they are 8 quarters for an expansionary one. Those of wage growth are 6 and 4 quarters, respectively. There is not clear asymmetry in the half-lives of quantity variables.

Table 5: Degree of asymmetry to different size of shocks

ϵ_0	π^w	π^p	MC	Y	H	i	r	β
0.5 S.D.	0.80	0.83	0.62	1.05	1.05	0.89	0.94	1.00
1.0 S.D.	0.65	0.70	0.40	1.11	1.11	0.80	0.89	1.00
1.5 S.D.	0.56	0.62	0.34	1.16	1.16	0.74	0.86	1.00
2.0 S.D.	0.49	0.57	0.35	1.20	1.20	0.70	0.83	1.00
2.5 S.D.	0.46	0.50	0.31	1.24	1.24	0.64	0.78	1.00

To quantify the degree of asymmetry, I introduce the following measure:

$$Asym(y, k, \epsilon_0) = \sum_{t=1}^{k} |GIR(y, t, \epsilon_0)| / \sum_{t=1}^{k} |GIR(y, t, -\epsilon_0)| \tag{48}$$

where y is the target variable and ϵ_0 is the initial exogenous shock with $\epsilon_0 > 0$. k is the time-horizon, which is set at 4. The measure compares relative size of the responses to a contractionary shock. Table 5 shows the degree of asymmetry is increasing in the size of shock. For example, the response of the price inflation to a 2.5 S.D. contractionary shock is smaller by 50 percent than the response to an expansionary shock with the same magnitude, while the difference is only 17 percent for a 0.5 S.D. shock.

6.3 Simulated Phillips curve

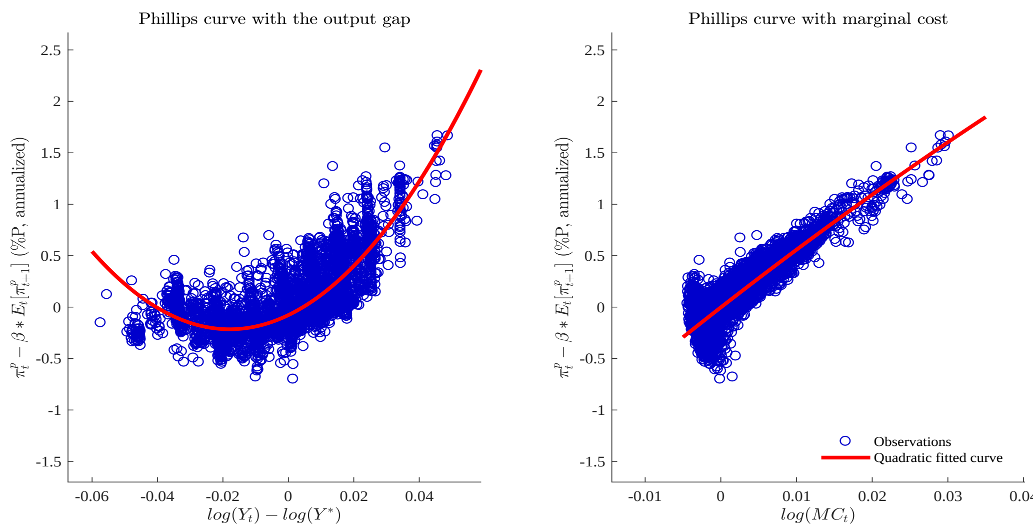
In Figure 5, I simulate the model economy to plot the two representations of the Phillips curve. The output gap representation of the NKPC in the left panel becomes flatter when the output gap takes negative values, and the quadratic fitted curve exhibits strong convexity. This is exactly because the binding DNWR constraint create a wedge between the output gap and marginal cost and the wedge shifts up the NKPC in recessions. Since the effect is increasing in the size of shocks, the Phillips curve looks downward sloping on its left tale. On the other hand, the marginal cost representation of the Phillips curve in the right panel stays almost linear. It reflects the fact that the firms' price setting behavior given a level of marginal cost does not change over business cycles in the model, though small deviations can emerge due to the fluctuations of the stochastic discount factor.

7 Discussion and relation to literature

This section provides discussion on the key mechanism of the model and its relation to the existing literature.

Shifts of cross-sectional wage distribution. A key feature to understand the non-linear dynamics of the model is shifts of cross-sectional distribution. Figure 6 shows the cross-sectional

Figure 5: Simulated Phillips curve



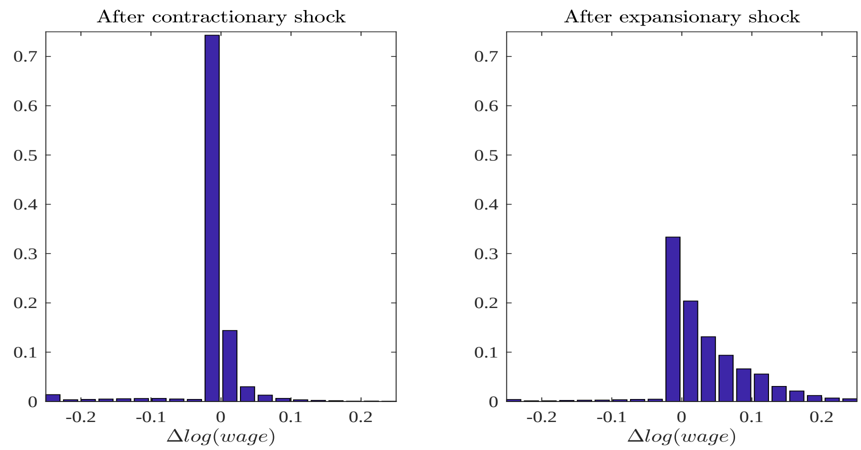
Notes: Each panel displays scatter plots of the simulated data and the quadratic fitted curves of it. Since we discretize the state space with the relatively small number of exogenous states, we use interpolation between observations to generate a smooth transition path between states. Specifically, we generated a time-series of data for 5,000 periods on the discretized state space and interpolated at the middle point of exogenous states using the linear interpolation. The total number of observations is therefore 9,999 in each panel.

distribution of individual wage changes when the economy is hit by contractionary and expansionary discount factor shocks. The size of spike at zero wage changes indicates that a larger fraction of workers is stuck at the DNWR constraint upon a contractionary shock. The observation is consistent with the micro evidence of Daly and Hobijn (2014) and Fallick et al. (2016), who document that the fraction of workers with zero wage changes substantially increased after the Great Recession. That leads to a stronger downward sluggishness of the aggregate wage than upward.

Another important feature is the role of idiosyncratic shocks. The calibrated parameter values identify the standard deviation of idiosyncratic shocks is much larger than that of aggregate shocks. Consequently, non-trivial fraction of workers experiences wage increases even after a contractionary aggregate shock because the effects of their positive idiosyncratic shock exceed the contraction of the aggregate economy. On the other hand, downward wage adjustment is truncated at zero as long as workers are subject to the DNWR constraint. Thus, the Jensen's inequality implies that the average wage change becomes higher than the case without idiosyncratic shocks. That further impedes the adjustment of the aggregate wage upon a contractionary shock. It would be worth pointing that the importance of idiosyncratic shocks is emphasized in the literature of price stickiness as well. For instance, Nakamura and Steinsson (2010) argue that, in their calibrated menu cost model, idiosyncratic shocks are large enough so that firms react to idiosyncratic shocks rather than aggregate shocks, which results in a substantial degree of aggregate price stickiness.

Wage markup. The wage markup in my model is closely related to that of Erceg et al. (2000),

Figure 6: Cross-sectional distribution of individual wage changes upon aggregate shocks



Christiano et al. (2005), and many others who incorporate the staggered contract of Calvo (1983) into wage settings. In their models, the wage rigidity introduces a time-varying wage wedge between real wage and the marginal rate of substitution between consumption and hours worked as well. However, my model is distinguished from these studies in several important dimensions. First of all, the fluctuations of the wage markup of the model depend on the sign of an exogenous shock, showing significant asymmetry in booms and recessions. Intuition is obtained by individual labor market equilibrium shown in Figure 7. In the left panel, the negative income effect reduces the marginal rate of substitution upon a contractionary shock. However, since nominal wage reductions are prevented by the DNWR constraint (I suppose that the price level is constant upon a shock in this partial equilibrium analysis), the wage markup should increase so that the labor supply curve shifts up to meet the labor demand curve. In the right panel, in contrast, the wage markup does not respond to an expansionary shock as long as the DNWR constraint does not bind.

The fluctuations of the wage markup depend on the size of an exogenous shock as well. To this regard, an exogenous shock has two effects on the wage markup. The direct effect is that an exogenous shock affects the individual wage markups for the workers whose DNWR constraint is already binding. In addition, an exogenous shock changes the portion of workers with and without the binding constraint by changing their desired wages. Therefore, the total effect is increasing in the size of shocks. In contrast, the staggered contract of Calvo (1983), in which a constant fraction of workers faces with the constraint in each period, lacks the second effect and therefore does not generate significant non-linearity of the markup dynamics.

Markup shock to the NKPC. King and Watson (2012) point out that a medium scale DSGE model often requires sizable and frequent exogenous markup shocks to the NKPC to account for the actual inflation. In line with their finding, Del Negro et al. (2015) argue that a large positive

Figure 7: Asymmetry of wage markup fluctuations

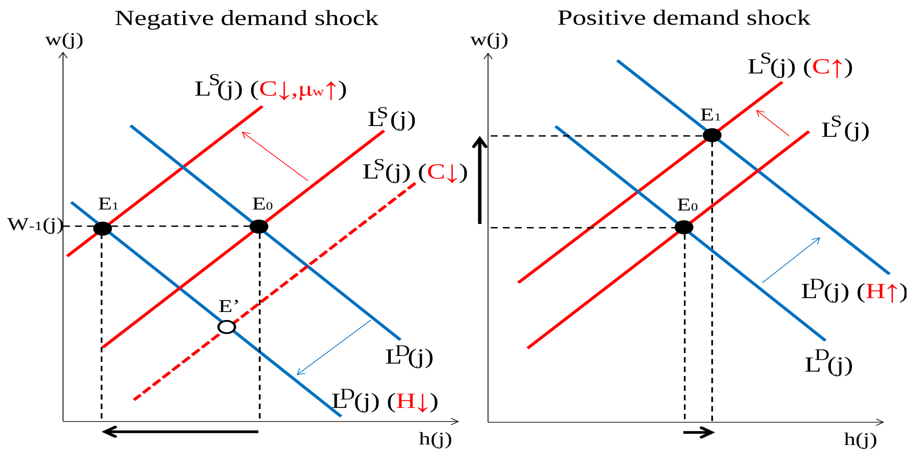
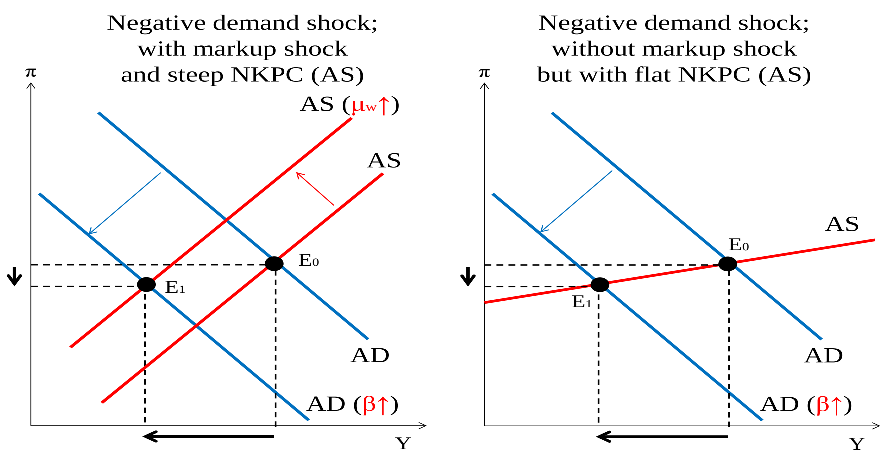


Figure 8: Demand and markup shocks with different slope of the NKPC



Notes: This figure corresponds to FIGURE 5 of Del Negro et al. (2015).

markup shock should be in company with a negative demand shock to answer the missing deflation puzzle under a relatively steep NKPC. They instead propose that a sufficiently flat Phillips curve as in the right panel of Figure 8 can address the puzzle. To this regard, my model generates a rise in the wage markup endogenously upon a negative demand shock through DNWR. The rise of the wage markup shifts up the NKPC (the AS curve), as a consequence of which the decline of inflation is moderate despite a large shift of the AD curve as shown in the left panel of Figure 8.

Implication to anchoring inflation expectations. Several studies emphasize the fact that the inflation expectation was stable during and after the Great Recession to address the missing deflation puzzle. Some of them attribute it to the departure from the full information and rational expectation (FIRE) model (Coibion and Gorodnichenko (2015)) or discrete regime changes of the economy (Bianchi and Melosi (2017)). On the other hand, I argue that the stable inflation expectations are consistent with my model although I stick to a FIRE model without regime switching. To see this point, iterating the linearized version of the NKPC (36) forward yields

$$\mathbb{E}_t[\pi_{t+1}] = \kappa \mathbb{E}_t \left[\sum_{s=0}^{\infty} D_{t+s+1} \hat{MC}_{t+s+1} \right] \tag{49}$$

(49) implies that the inflation expectation $\mathbb{E}_t[\pi_{t+1}]$ is the infinite sum of the discounted values of the future marginal costs. Therefore, the model potentially address the stable inflation expectations as long as the future marginal costs are sufficiently stabilized. I show that the model indeed predicts a moderate decline of the inflation expectations that matches the survey based inflation expectations in the data after the Great Recession in a counterfactual analysis in Section 8.

Comparison with different specifications of wage adjustment. In Appendix D, I compare the dynamics of models with different specifications of wage adjustment. Specifically, I solve and calibrate a flexible wage model and a quadratic wage adjustment cost model as well as the baseline model with DNWR. It should be noticed that the quadratic wage adjustment cost model coincides with the Calvo-type staggered contract model in the first order, which is widely used in the existing New Keynesian literature. I calibrate the parameter for wage stickiness according to the micro evidence reported by Barattieri et al. (2014). Other parts of the models than wage adjustment are identical to the baseline model. Figure D.4 compares the GIR in different models. In the flexible wage model, wage growth, marginal cost, and price inflation respond strongly to a discount factor shock. Since the effects of an exogenous shock are absorbed by adjustments of price variables, quantity variables such as output and consumption do not react a lot. On the other hand, the quadratic wage adjustment model generates moderate responses in price variables and sizable responses of quantity variables. However, there are several important differences from the model with DNWR. First of all, the quadratic adjustment cost model does not bring about significant asymmetry since the wage adjustment cost is symmetric by construction.[11] As a result, the inflation responses to a 2 S.D. contractionary discount factor shock is almost twice as large as those in the model with DNWR, while the responses to an expansionary shock with the same magnitude is slightly smaller. Second, the propagation of an exogenous shock is not as stringent as the model with DNWR. The half-lives of wage growth and price inflation to a 2 S.D. contractionary shock are 2 and 7 quarters in the quadratic wage adjustment cost model, whereas they are 6 and 10 quarters in the model with DNWR. The difference reflects the state-dependency of DNWR. After a contractionary shock, for instance, workers does not react to improvements of the state of the economy at all as long as the DNWR constraint binds, whereas the quadratic adjustment cost model allows for gradual responses in each period. Further discussion on the state-dependency of the model is provided in Section 8.

[11]Since the model is solved by a global method, non-linearity can arise from other parts of the model than the wage adjustment such as the curvature of the utility function. However, I find that such non-linearity is quantitatively small.

I also compare the baseline model with a model that embeds asymmetric wage adjustment costs, because the class of model potentially generates non-linearity in aggregate dynamics. In the literature, Kim and Ruge-Murcia (2009) and Fahr and Smets (2010) use an asymmetric wage adjustment cost function to approximate DNWR. More recently, Aruoba et al. (2017) augment the model to include both of asymmetric wage and price adjustment cost to find that the model can capture the non-linearity in the data well. I calibrate an asymmetric wage and price adjustment cost model based on the estimated parameters of Aruoba et al. (2017).[12] Figure D.6 compares the GIR of key variables to different sign and size of discount factor shocks. Interestingly, the non-linearity of price inflation and output are quite similar in the two models. However, it is worth pointing that the model with DNWR is calibrated consistently with the frequency of individual price changese in micro data, whereas Aruoba et al. (2017) identify a much higher parameter value for the degree of price stickiness (flatter Phillips curve). Moreover, the responses of wage growth and real wage display much stronger non-linearity in the model with DNWR. This is because the non-linearity of the model with DNWR stems from asymmetric individual wage adjustments, whereas Aruoba et al. (2017)'s estimates indicate strong asymmetry in price adjustment rather than wage adjustment. To this regard, I point out that the model with DNWR matches the moderate decline of wage growth and real wage after the Great Recession fairly well in a counterfactual analysis in Section 8. However, further investigation on the comparison of different models is left for future research.

Connection to the literature of the micro evidence on DNWR. A crucial assumption of my model is the DNWR constraint for individual wage settings. Hence, I quickly review the literature about the micro evidence on DNWR to assess the validity of the assumption. More comprehensive literature review is found in Basu and House (2016). McLaughlin (1994) is one of the earliest studies to test for the presence of DNWR using individual wage data in the U.S. Though his result was not favorable for DNWR, subsequent studies found evidence of it. For instance, Card and Hyslop (1997), using the individual wage data in the CPS, find a large spike at zero in wage change distribution. Lebow et al. (1995) report asymmetry of wage change distribution arising from DNWR, and Kahn (1997) propose a formal test to verify the asymmetry of distribution. To elaborate their findings, Gottschalk (2005) corrects for a measurement error problem in self-reported data by using an econometric strategy that detects structural breaks. Barattieri et al. (2014) employ his technique to find that nominal wage reductions correspond only to 12.3 percent of non-zero nominal wage changes in the SIPP during 1996-1999 after correcting for measurement errors.

One might be concerned about the possibility that benefits such as bonus, pensions, and other supplementary payments are used to adjust the total compensation of workers even if wages are downward rigid. To this end, Kurmann and McEntarfer (2017) report that, using administrative

[12] I use the posterior mean for the sample of 1960Q1-2007Q4 reported by Aruoba et al. (2017). Details are provided in Appendix D.

worker-firm linked data in Washington state, the spike at zero of the changes in the average hourly compensation (sum of wages and benefits) is much smaller than that of wages only, and declines of compensation are not rare in individual data. They claim that their results are less affected by measurement errors than the studies using self-reported data since they use administrative data. On the other hand, Lebow et al. (2003), computing changes of wages and benefits separately in the ECI, document that, although benefits change more frequently than wages, the changes of benefits are not systematically related to wage changes. Based on these observations, they conclude that the hypothesis that benefits are used to offset nominal wage rigidities is not supported in the data. [13]

Another interesting finding in the empirical literature is international differences in the degree of DNWR, though this paper exclusively focuses on the inflation dynamics in the U.S.[14] For instance, Smith (2000) investigate weekly average compensation in the U.K. during 1991-1996 to report that only 1 percent of workers are constrained by DNWR after correcting measurement errors and long-term contracts. Elsby et al. (2016) report that the degree of DNWR in the U.K. is weaker than the U.S. using a longer time-series of data and argue that the downward flexibility of nominal wages in the U.K. resulted in the relatively rapid adjustment of real wage after the Great Recession. On the other hand, a sequence of studies by Kuroda and Yamamoto (2003) and Kuroda and Yamamoto (2014) document that, after the financial crisis in the late 1990s of Japan, DNWR disappeared from the individual wage data in Japan although it was present until the mid 1990s. The consequence of these cross-country variations in the degree of DNWR for the inflation dynamics of each country would be an interesting research question though it is above the scope of this paper.

8 Modeling the Great Recession

This section adds several extensions to the baseline model to investigate whether the model can account for the inflation dynamics during and after the Great Recession.

8.1 ZLB

A number of studies argue that the ZLB of the nominal interest rate is an essential element to understand the Great Recession (Christiano et al. (2015), Basu and Bundick (2017), Aruoba et al. (2018), etc). In this subsection, therefore, I introduce the ZLB into the baseline model to explore its implications.

[13] It should be noted that the unit of observation of the ECI is jobs instead of workers. How much their empirical results are affected by the difference in the unit of observations would be a subject of future research.

[14] In terms of wage rigidities in general, Dickens et al. (2007) find, in the International Wage Flexibility Project, the degree of nominal and real wage rigidity significantly varies across countries.

As a benchmark, I assume that the central bank follows a Taylor (1993) rule with the ZLB:

$$R_t^d = R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{50}$$

$$R_t = max\{\ R_t^d\ ,\ 1\ \} \tag{51}$$

Although most of the previous studies that investigate the role of the ZLB after the Great Recession assume the standard Taylor rule in (50),[15] the Federal Reserve announced a commitment to keep the low interest rate policy when they faced with the ZLB. To take into account the effect of this type of forward guidance, I consider the history-dependent rule proposed by Reifschneider and Williams (2000):

$$R_t^d = R^* \left(\frac{R_{t-1}^d}{R_{t-1}}\right) \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{52}$$

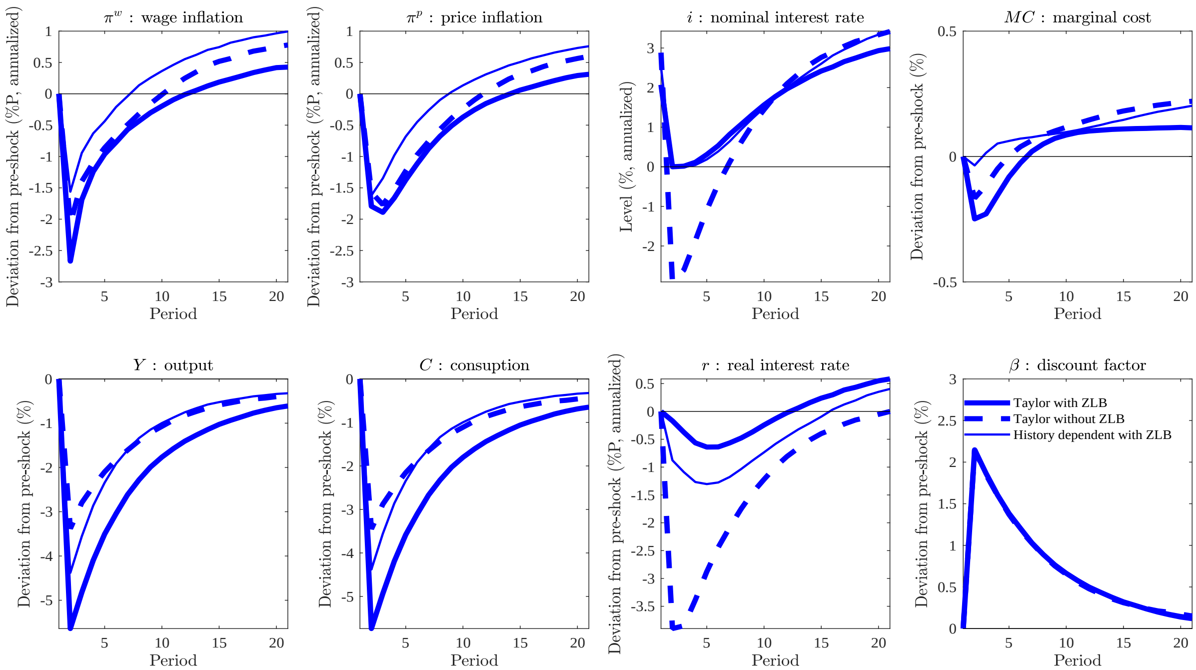
$$R_t = max\{\ R_t^d\ ,\ 1\ \} \tag{53}$$

Under the history-dependent rule, the central bank keeps track of the past interest rate gap, that is, the difference between the desired interest rate implied by (52) R_{t-1}^d and the actual rate R_{t-1}. Once the nominal interest rate is constrained at the ZLB, the central bank continues the low interest rate policy until the gap is cleared even if the interest rate implied by the standard Taylor rule becomes positive. Though several ways to implement forward guidance in a general equilibrium model has been studied in the literature (Eggertsson and Woodford (2003), Del Negro et al. (2012), McKay et al. (2016), for example), the history-dependent rule is distinguished from others in that it is fully embedded in the rational expectation equilibrium as a monetary policy rule. Basu and Bundick (2015) argue that the history-dependent rule has desirable properties to remove the contractionary bias of the ZLB, that is, the bias that the central bank charges higher interest rate than the desired one on average over business cycles in the presence of the ZLB.

Figure 9 displays the GIR under different monetary policy rules with and without the ZLB. Several aspects of the results are noteworthy. First, in line with previous studies, the ZLB has an amplification effect to a demand shock. Second, the amplification effect in the model works much more strongly for quantity variables rather than price variables. For instance, the responses of price inflation are larger by around 20 percent in the presence of the ZLB, while the output responses are amplified by more than 50 percent. Economic intuition behind the results is as follows. At the ZLB, a contractionary demand shock raises real interest rate due to the lack of the offsetting monetary policy responses, and it reduces consumption through the Euler equation. In a frictionless labor market, the decreased marginal rate of substitution due to the negative income effect would lead to a decline of real wage, which in turn would reduce inflation through the NKPC. In the presence of DNWR, however, the decline of real wage is hindered by the binding DNWR constraint, while the dampened response of real wage is compensated by a further contraction

[15] Few exceptions include Basu and Bundick (2015) and Katagiri (2016).

Figure 9: GIR with and without the ZLB



Notes: Each panel shows the GIR to a 3 S.D. contractionary discount factor shock. The x-axes represent the time horizon after the initial shock. The y-axes are in terms of the deviation from the value before the shock except for the nominal interest rate, which is shown in the level. Models with different monetary policy rules are solved under the same parameter values.

of hours worked as a consequence of the labor market equilibrium. Third, under the history-dependent rule, the amplification effect of the ZLB is partly offset because the commitment to the future low interest rates affects the current consumption through the forward looking nature of the Euler equation.

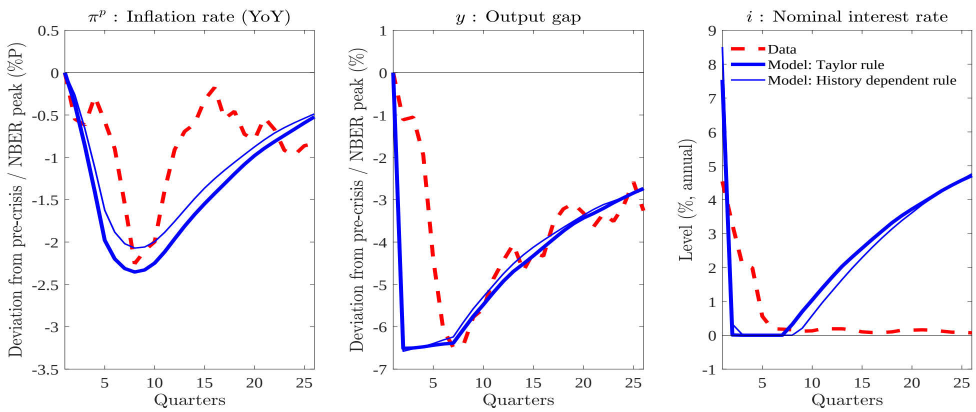
8.2 Counterfactual for the Great Recession

This subsection conducts a counterfactual analysis of the Great Recession. To calibrate the model to U.S. data during the Great Recession, I borrow the concept of "severe recession" proposed by Krueger et al. (2016). They define severe recession as the periods in which unemployment rate hits 9 percent at least in one quarter and stays above 7 percent. In the post-war U.S. economy, 1980Q2-1986Q2 and 2009Q1-2013Q2 satisfy these criteria. I employ their calibrated values for state transition probabilities:

$$P_\beta = \begin{bmatrix} P_{L\to L} & P_{L\to H} \\ P_{H\to L} & P_{H\to H} \end{bmatrix} = \begin{bmatrix} 0.9910 & 0.0090 \\ 0.0455 & 0.9545 \end{bmatrix} \tag{54}$$

where the high discount factor state denoted by H corresponds to the severe recession. Notice that the expected duration of the severe recession is much shorter (around 22Q) than that of the normal state (around 111Q).

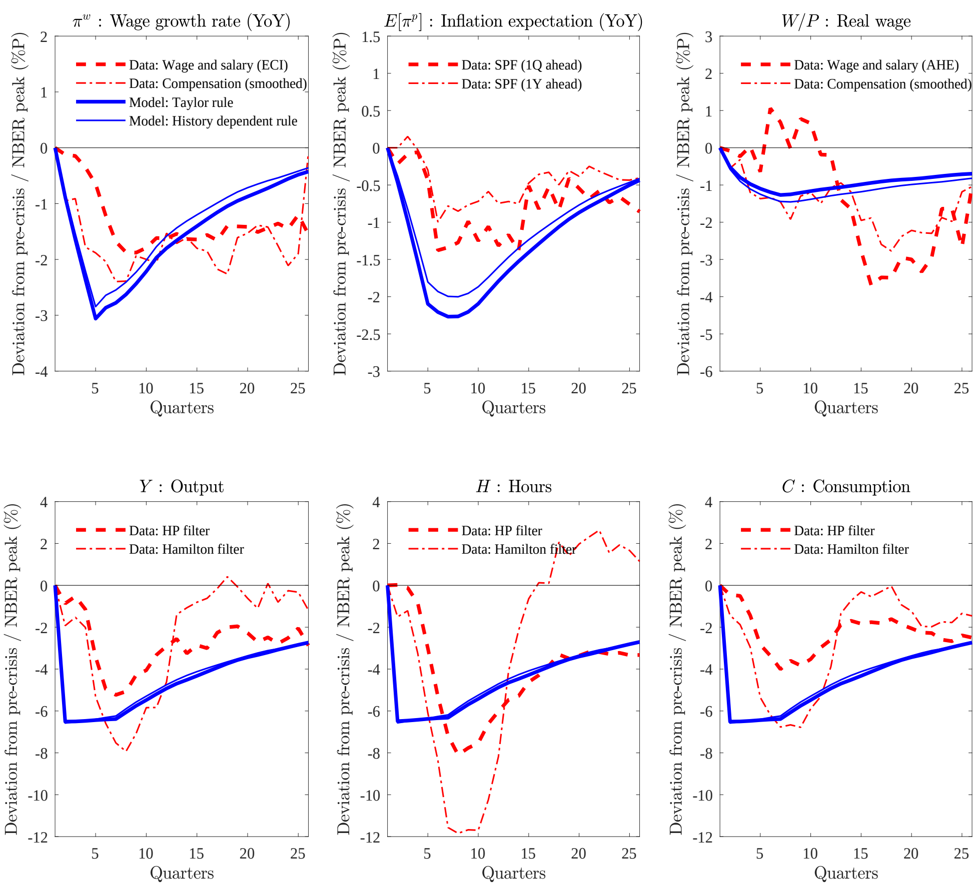
Figure 10: Counterfactual for the Great Recession (1)



Notes: Each panel shows the GIR to exogenous 6 quarter consecutive severe recession shock after 24 quarters of boom periods. The length of shocks and boom periods correspond to those of the Great Recession (expansion: 2002Q1-2007Q4, contraction: 2008Q1-2009Q2). The nominal interest rate in the data is the effective federal funds rate. The definition of other variables is the same as Figure 1.

Figure 10 and Figure 11 display the counterfactual to the contractionary shock that replicates the Great Recession. The size of the shock is calibrated to match the drop of the output gap in the data in each model. In Figure 10, the calibrated Great Recession shock only leads to 2.4 percentage point decline of the year-on-year inflation rate under the standard Taylor rule with the ZLB. This quantitative result is comparable with the U.S. economy during the Great Recession, in which the actual inflation rate in the GDP implicit price deflator declined by 2.3 percentage point from the peak to the bottom (2007Q4:2.5 percent→2009Q3:0.2 percent). The model replicates the sluggish recovery of inflation in the data as well. Interestingly, the magnitude of the decline of inflation under the history-dependent rule is 2.1 percentage point, which is quite close to that of the standard Taylor rule. It implies that once the magnitude of the output gap drop is taken as given the two monetary policy rules yield similar relative responses of inflation to the output gap, even though the history-dependent rule offsets overall amplification effects of the ZLB as is found in the previous subsection. In Figure 11, the model replicates the dynamics of other variables fairly well. In particular, the data suggests that both of wage and compensation declined moderately after the Great Recession and the model captures these movements with the difference in the magnitude of the responses from the data being less than 1 percentage point. It is also notable that the model is consistent with the stable inflation expectation in the data since dampened responses of marginal cost due to DNWR prevent the inflation expectations from declining. As for real quantities, though the model cannot perfectly replicate the relatively large drop of hours worked and small decline of consumption in the data, they are presumably because I abstract capital investment, which is not the main focus of this paper. It is worth pointing that the model does not have a number of ingredients that previous studies argue are important to account for the missing deflation such as high degree of price stickiness, exogenous shocks to the inflation expectations, and financial frictions. Instead, the only extension from the 3-equation New Keynesian model presented in Section 3 is the presence of the DNWR constraint for individual workers and the ZLB, but still the model succeeds in accounting for the key moments regarding the missing deflation puzzle.

Figure 11: Counterfactual for the Great Recession (2)

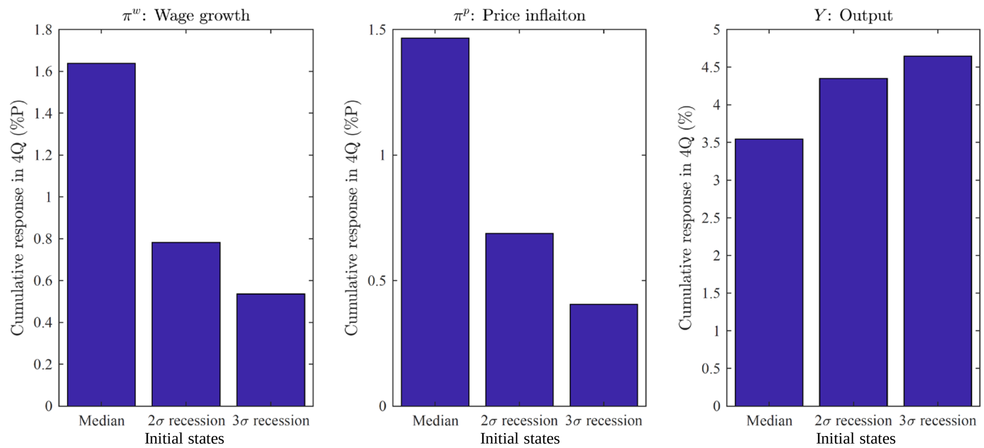


Notes: The definition of GIR is the same as Figure 10. Data is in the deviation from the business cycle peak before the Great Recession defined by the NBER (2007Q4). In the top panels, wage growth π^w is the wage and salary in the Employment Cost Index and the compensation per hour in the non-farm business sector. The compensation series is smoothed as five quarters centered moving average. Inflation expectation $E[\pi^p]$ is the median forecast for one quarter and four quarters ahead GDP deflator in the Survey of Professional Forecast. The one quarter ahead forecast is annualized. For real wage W/P, price index is the GDP implicit price deflator. Wage is the average hourly earnings of production and non-supervisory employees in the private sector and the compensation per hour in the non-farm business sector. Real wage series is detrended by the HP filter. In the bottom panels, output Y is the real GDP, hours H is the total hours in the non-farm business sector, consumption C is the real personal consumption expenditures. Each series is taken log and detrended by the HP filter and the Hamilton filter.

8.3 Implication to the excessive disinflation

This subsection investigates the implications of the model to the excessive disinflation after the Great Recession. Figure 12 shows the GIR to a 1 S.D. expansionary shock from different initial states. The model displays significantly divergent responses in each initial state. Starting from a 3 S.D. recession state, which corresponds to the severe recession state, the positive responses of

Figure 12: State dependency of GIR



Notes: Each bar shows the cumulative responses in the four quarters after a 1 S.D. expansionary discount factor shock from different initial states. To draw each initial state, I simulate the model economy randomly for ten quarters after the median state, 2 S.D. recession state, and 3 S.D. recession state, respectively.

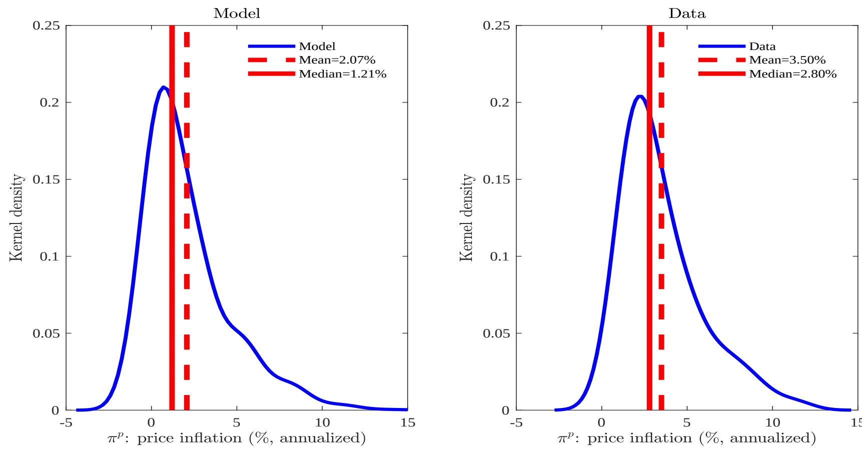
wage growth and price inflation are roughly three times smaller than those from the median state. This is due to the state-dependent nature of DNWR. Upon a severe recession shock, workers' desired wages decline due to the negative income effect and fall short of their actual wages since the DNWR constraint binds. In a recovery phase, even when their desired wages start to rise as the state of the economy improves, workers never raise their actual wages as long as the DNWR constraint binds. This mechanism delays the recovery of wage growth, which in turn leads to a slow recovery of inflation through sluggish rises of marginal cost. On the other hand, the recovery of output is relatively fast from a severe recession, but the quantitative result suggests that the differences of the output responses are not as significant as wage growth and inflation.

Lastly, Figure 13 shows the distribution of price inflation in time-series simulation of the model and the data. In the model, the distribution of inflation is positive skewed because of the asymmetric effect of DNWR. Consequently, the median inflation rate (1.21 percent) is substantially lower than the 2 percent of the calibrated target rate in the Taylor rule, while the mean inflation rate (2.07 percent) is almost around the target rate. The result indicates that lower inflation rate than the target level is more likely to realize in each period even if the target rate is achieved in the mean. This finding might be counterintuitive, but is indeed consistent with a wide class of the Taylor-type monetary policy rule. To see this point, taking the the unconditional expectation of the Taylor rule (38) leads to:

$$log(\mathbb{E}[R_t]) - log(R^*) \cong \delta_\pi(log(\mathbb{E}[\Pi_t]) - log(\Pi^*)) + \delta_y(log(\mathbb{E}[Y_t]) - log(Y^*)) \tag{55}$$

Notice that the equation does not strictly hold because I ignore the Jensen's inequality terms. Equation (55) implies that inflation is stabilized in the mean under the Taylor rule, although the equation does not guarantee the stabilization of the median inflation to the target rate.

Figure 13: Distribution of inflation



Notes: The kernel density estimator is computed from simulated and actual data. In the model, I simulate the model economy for 51,000 periods and discard the initial 1,000 observations. Data is the quarter on quarter growth rate of the GDP implicit price deflator. Sample period is from 1955Q1 to 2007Q4. The end of the sample period is determined to exclude the ZLB periods.

9 Conclusion

In this paper, I introduce DNWR for individual workers into an otherwise standard New Keynesian DSGE model. DNWR accounts for the flattening of the observed Phillips curve relationship between inflation and the output gap, while keeping the marginal cost representation of the NKPC unchanged. The endogenous shifts of cross-sectional wage distribution generate non-linear dynamics in many dimensions including the sign-, the size-, and the state-dependency of the consequence of an exogenous shock. I demonstrate that the calibrated model successfully matches the key moments of the inflation dynamics during and after the Great Recession, which are often referred to as the missing deflation and the excessive disinflation.

A number of extensions are possible for future research. First, assessing other dimensions of aggregate dynamics through the lens of my model is a natural extension, though I consider the simplest setting to investigate inflation dynamics in this paper. For instance, incorporating unemployment is one promising option. Exploring the interaction between the heterogeneity of labor and that of consumption would be interesting as well. To this regard, though recent papers such as Hall (2017) identify that the movements of discount factor are essential to explain the aggregate dynamics after the Great Recession, dealing with consumption heterogeneity might help one to reconcile the large fluctuations of discount factor as Guerrón-Quintana (2008) suggests. Second, it would be worthwhile investigating optimal monetary policy in the economy with DNWR. Though Kim and Ruge-Murcia (2009) and Coibion et al. (2012) study optimal policy in a representative agent framework, taking into account heterogeneity might change welfare implications. Moreover, although the state of the economy is characterized by cross-sectional distribution in a heterogeneous agent setting, it is in practice difficult for the central bank to keep track of the distribution in a timely manner. Therefore, how to approximate the optimal policy as an implementable policy rule would be a valuable question as well. Lastly, on the empirical side, my model yields a number of testable implications. In particular, it would be beneficial to explore how much the model can account for the evolution of cross-sectional wage distribution after the Great Recession in more detail.

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Online Appendix: Downward Nominal Wage Rigidity and Inflation Dynamics during and after the Great Recession

Tomohide Mineyama

A Empirical Evidence

A.1 Construction of marginal cost

Specification of production function. For a baseline case, I assume the Cobb-Douglas production function with overhead labor (CDOH):

$$Y_t = F(H_t, K_t, Z_t) = \{Z_t(H_t - \bar{H})\}^{\alpha} K_t^{1-\alpha} \tag{A.1}$$

where Y_t denotes output, Z_t labor augmenting technology, H_t labor inputs, K_t capital inputs. $\bar{H}$ is the overhead component of labor inputs that is not directly linked to value added production. The first order condition (FOC) for labor inputs implies

$$\theta_t^H \equiv \frac{\partial log(F_t)}{\partial log(H_t)} = \alpha \frac{H_t}{H_t - \bar{H}} \tag{A.2}$$

Using (2.8), the FOC is rearranged to the specification of marginal cost:

$$MC_t^{CDOH} = \frac{1}{\alpha}\left(1 - \frac{\bar{H}}{H_t}\right) s_t^H \tag{A.3}$$

with $s_t^H = \frac{W_t H_t}{P_t Y_t}$ being the labor share. The specification leads to

$$\hat{MC}_t^{CDOH} = \frac{\bar{H}/H^{ss}}{1 - \bar{H}/H^{ss}} \hat{H}_t + \hat{s}_t^H \tag{A.4}$$

where $\hat{x}$ denotes the log-deviation from the steady state. For parameterization, I borrow the estimated value of Basu (1996) to calibrate $\bar{H}/H^{ss} = 0.288$. The value is in line with other estimates in literature such as 0.20 of Ramey (1991) and 0.14 of Bartelsman et al. (2013). Bartelsman et al. (2013) point out as a reference that in the U.S. manufacturing industries non-production workers compose of roughly 30 percent of total employment and managers of 10 percent.

For robustness check, I consider alternative specifications: the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES). The Cobb-Douglas production function is given by:

$$F(H_t, K_t, Z_t) = (Z_t H_t)^{\alpha} K_t^{1-\alpha} \tag{A.5}$$

The FOC for labor inputs formulates marginal cost to be proportional to the labor share:

$$MC_t^{CD} = \frac{1}{\alpha} s_t^H \tag{A.6}$$

and

$$\hat{MC}_t^{CD} = \hat{s}_t^H \tag{A.7}$$

Under a CES production function,

$$F(H_t, K_t, Z_t) = \left\{ \alpha \left(Z_t H_t \right)^{\frac{\nu-1}{\nu}} + (1-\alpha) K_t^{\frac{\nu-1}{\nu}} \right\}^{\frac{\nu}{\nu-1}} \tag{A.8}$$

we obtain

$$MC_t^{CES} = \frac{1}{\alpha} \left(\frac{Y_t}{Z_t H_t} \right)^{\frac{\nu-1}{\nu}} s_t^H \tag{A.9}$$

and

$$\hat{MC}_t^{CES} = \frac{\nu - 1}{\nu} (\hat{Y}_t - \hat{Z}_t - \hat{H}_t) + \hat{s}_t^H \tag{A.10}$$

where ν represents the elasticity of substitution between labor and capital inputs. I follow Gali et al. (2007) to calibrate $\nu = 0.5$. For the series of Z_t, I use the utilization-adjusted quarterly-TFP for the U.S. business sector constructed based on Fernald (2014).

Detrending. An important issue when using the series of the labor share in U.S. data is detrending, because the data displays a low frequent downward trend. Though there is substantial debate regarding the causes behind the trend, many of existing studies attribute it to structural changes of the economy such as offshoring of manufacturing industries and declining relative price of investment goods due to advances in information technology, or others point out mismeasurement of data (Elsby et al. (2013), Karabarbounis and Neiman (2013), etc). Since the main focus of this paper is on business cycle fluctuations related to inflation dynamics, I use a filtering method to extract cyclical components of the labor share. Similar methods are employed by Mavroeidis et al. (2014) when they estimate the NKPC. Specifically, each series of marginal cost is detrended by the Hamilton filter. Hamilton (2017) argues that the Hamilton filter has desirable time-series properties compared to the HP-filter, which is widely used in business cycle analysis. In particular, the one-sided method of the Hamilton filter addresses the end of sample problem of the HP-filter.

A.2 Robustness checks for the estimation of the NKPC

A.2.1 Robustness check (1): alternative specifications of marginal cost

Table A.1 reports the results of the OLS estimation of the NKPC (4) with alternative specifications of marginal cost: marginal cost based on the Cobb-Douglas production function (CD) in column 1-4 and a production function with constant elasticity of substitution (CES) in column 5-8. The estimation results display quite similar patterns to the baseline specification of the Cobb-Douglas production function with overhead labor (CDOH) in Table 1. That is, the coefficients of marginal cost remain stable after the Great Recession, and the interaction terms with the post Great Recession dummy are not statistically significant.

Table A.1: OLS estimation with alternative specifications of marginal cost

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Marginal cost (CD)			Marginal cost (CES)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.447***	0.485***	0.465***	0.419***	0.467***	0.441***
	(0.079)	(0.074)	(0.078)	(0.080)	(0.074)	(0.079)
π_{t-1}	0.548***	0.510***	0.529***	0.575***	0.527***	0.552***
	(0.080)	(0.074)	(0.079)	(0.082)	(0.075)	(0.081)
x_t	0.208**	0.167**	0.193**	0.170***	0.136***	0.148***
	(0.082)	(0.067)	(0.078)	(0.056)	(0.045)	(0.053)
$postGR_t \times \pi_t^e$			0.290			0.439*
			(0.219)			(0.226)
$postGR_t \times \pi_{t-1}$			-0.308			-0.404*
			(0.232)			(0.234)
$postGR_t \times x_t$			-0.120			-0.130
			(0.155)			(0.098)
Adjusted R^2	0.952	0.946	0.946	0.935	0.931	0.930
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

A.2.2 Robustness check (2): the purely forward looking NKPC

I estimate the following purely forward looking NKPC:

$$\pi_t = \beta \pi_t^e + \kappa x_t + e_t \tag{A.11}$$

Table A.2 reports the results of the OLS estimation of the purely forward looking NKPC (A.11). Similar to the hybrid NKPC in Table 1, the coefficient of the marginal cost is stable after the Great Recession, and the interaction term of the marginal cost and the dummy variable for the post-Great Recession period is not significant.

Table A.2: OLS estimation of the purely forward looking NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	1.048***	1.029***	1.045***	0.998***	0.996***	0.996***
	(0.023)	(0.024)	(0.023)	(0.024)	(0.023)	(0.023)
x_t	0.458***	0.229**	0.425***	0.162*	0.140**	0.138*
	(0.063)	(0.052)	(0.060)	(0.083)	(0.066)	(0.080)
$postGR_t \times \pi_t^e$			-0.126			0.00500
			(0.137)			(0.097)
$postGR_t \times x_t$			-0.444***			0.0112
			(0.099)			(0.128)
Adjusted R^2	0.947	0.936	0.941	0.934	0.930	0.929
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The sign of the coefficient of the unemployment gap is flipped for comparison purposes. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

A.2.3 Robustness check (3): the rational expectation assumption

I follow Gali and Gertler (1999) to assume the rational expectation for seeing robustness of the baseline result in terms of assumptions on the expectation formation. Using the rational expectation assumption, the expected inflation can be replaced with the realized inflation and the rational expectation error:

$$\mathbb{E}_t[\pi_{t+1}] = \pi_{t+1} + \tilde{e}_{t+1} \tag{A.12}$$

The NKPC is rearranged to:

$$\pi_t = \beta\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t + e_{t+1} \tag{A.13}$$

with $e_{t+1} \equiv \beta\tilde{e}_{t+1}$. Notice that the OLS estimator is biased because e_{t+1} might be correlated with π_{t+1}. Adopting the insight of Gali and Gertler (1999), therefore, I use lagged variables as instruments for π_{t+1} to derive the GMM estimator. To this end, any variables at and before period t are valid instruments, because the rational expectation error e_{t+1} is orthogonal to any variable in the information set at period t. The estimation result is reported in Table A.3. The coefficient of the unemployment gap is not significant in column 1 and 2, and weakly significant with a negative sign to in column 3. Although the result is inconsistent with the theory of the NKPC, it is in line with the findings of previous empirical studies. To be precise, Gali and Gertler (1999) and Sbordone (2002) obtain insignificant estimates for the coefficient of the output gap under the rational expectation assumption. More recently, Adam and Padula (2011) find that the coefficient of the output gap is significant only when a survey based expectation measure is used instead of the rational expectation assumption. On the other hand, I confirm the baseline result regarding the marginal cost representation of the NKPC. The coefficient of marginal cost is significantly positive and does not decline after the Great Recession.

Table A.3: GMM estimation under the rational expectation assumption

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.729***	0.700***	0.709***	0.704***	0.683***	0.692***
	(0.051)	(0.045)	(0.058)	(0.048)	(0.056)	(0.053)
π_{t-1}	0.261***	0.289***	0.277***	0.283***	0.304***	0.294***
	(0.051)	(0.046)	(0.058)	(0.049)	(0.056)	(0.053)
x_t	-0.0273	-0.0138	-0.0350*	0.129***	0.104***	0.111***
	(0.021)	(0.012)	(0.021)	(0.040)	(0.033)	(0.040)
$postGR_t \times \pi_{t+1}$			0.208			-0.00340
			(0.284)			(0.164)
$postGR_t \times \pi_{t-1}$			0.174			0.0632
			(0.152)			(0.166)
$postGR_t \times x_t$			0.210**			-0.150
			(0.090)			(0.147)
N of obs.	208	243	243	208	243	243

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The two step GMM with an HAC weight matrix is employed. Instruments are the first to forth lagged GDP implicit price deflator, marginal cost, labor share, output gap, wage growth rate, commodity price inflation, short-long term interest rate spread, post-Great Recession dummy. HAC corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1955Q1 to 2007Q4 for *Before GR* and from 1955Q1 to 2016Q3 for *Full sample*, respectively.

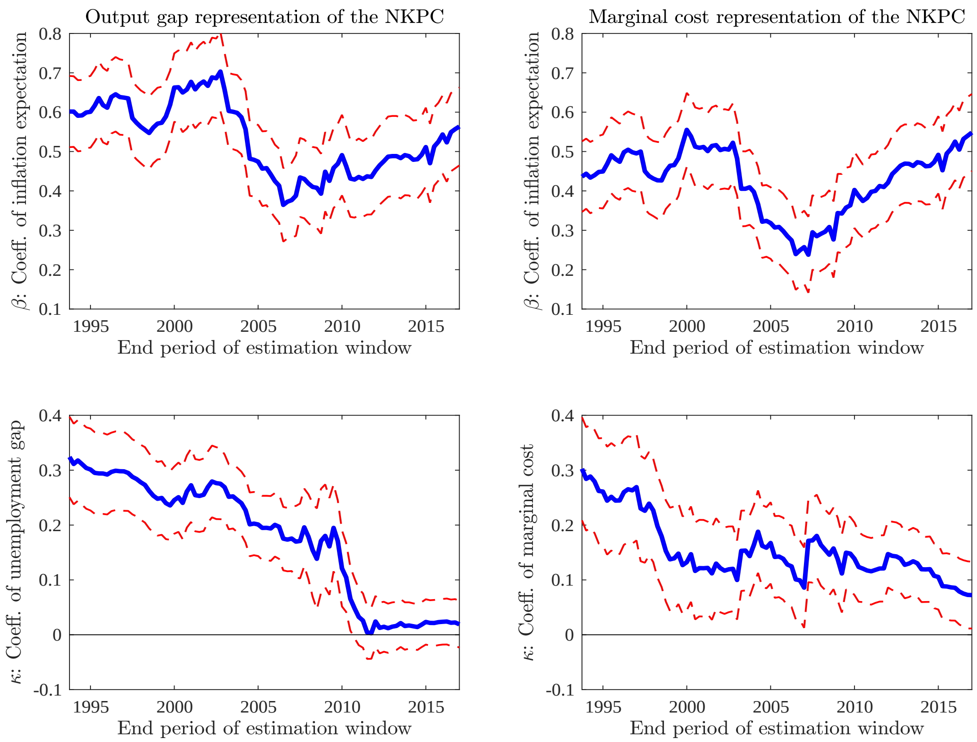
A.2.4 Robustness check (4): rolling OLS regression

To identify the specific timing of the changes of coefficients in the NKPC, I conduct a rolling OLS regression of the NKPC. I roll over 25 year window samples, each of which includes 100 observations. In order to save the number of parameters to estimate in a relatively small sample size, I impose a restriction $\beta + \gamma = 1$ as is often assumed in the literature such as Blanchard et al. (2015). The regression model for each rolling sample is given by:

$$\pi_t = \beta_T \pi_t^e + (1 - \beta_T)\pi_{t-1} + \kappa_T x_t + e_t, \quad for\ t \in [T - 99, T] \tag{A.14}$$

Figure A.1 shows the evolution of the estimated coefficients. The coefficient of the output gap shown in the left-bottom panel has a sharp drop around 2010 and stays around zero afterward. On the other hand, the coefficient of marginal cost in the right-bottom panel remains roughly constant around the period. The estimation result detects other interesting patterns in the time-variations of coefficients, such as a rise in the coefficients of the forward looking inflation term after mid 2000s and a slowly declining trend in the coefficient of the output gap and marginal cost in 1990s. However, they are beyond the scope of this paper.

Figure A.1: Rolling OLS regression of the NKPC with 25 year window



Notes: Solid line is the OLS estimator, while dashed line is the 68% confidence band. The definition of each variable is the same as Table 1.

A.2.5 Robustness check (5): Markov-switching model

One might be concerned that the result of the rolling regression reflects particular observations in each sample window. The Bayesian methods, on the other hand, make full use of the entire sample to specify the timings of parameter changes. Specifically, I estimate the Markov-switching model for the coefficients and the standard deviation of the error term in the NKPC:

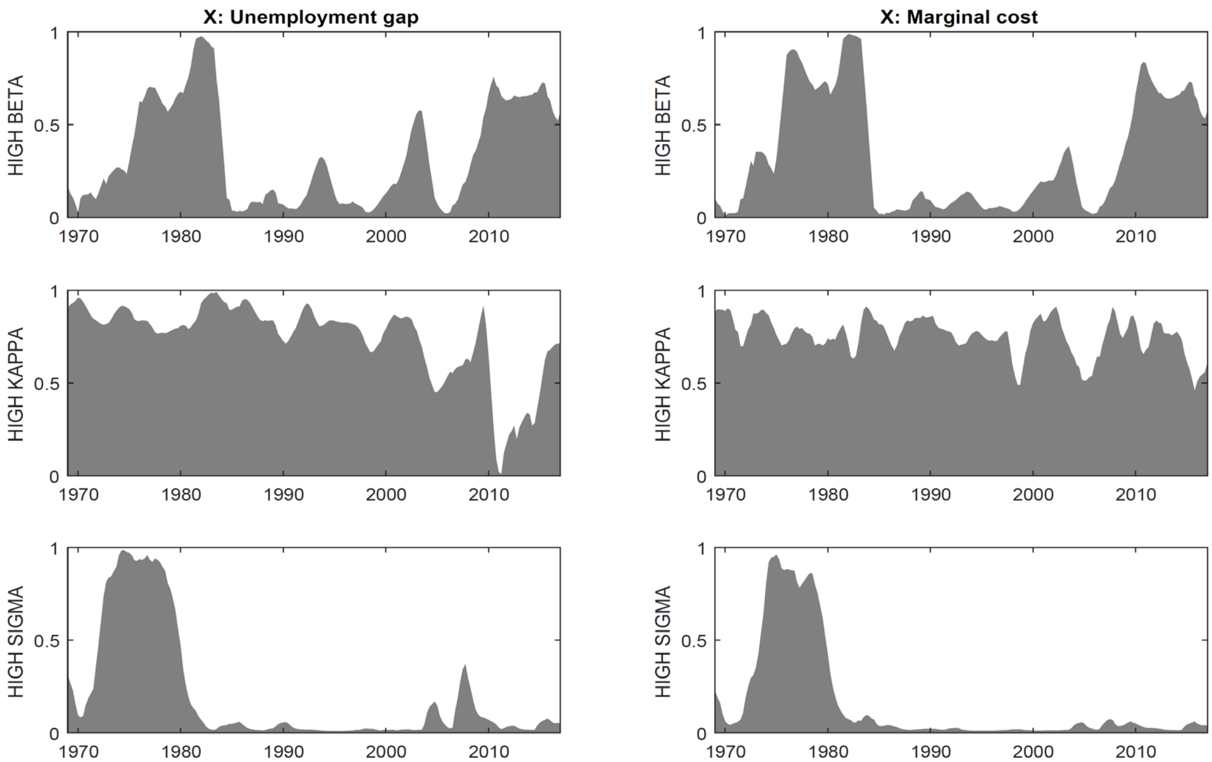
$$\pi_t = \beta(S_t)\pi_t^e + (1-\beta(S_t))\pi_{t-1} + \kappa(S_t)x_t + e_t, \quad e_t \sim N(0, \sigma(S_t)^2) \tag{A.15}$$

I consider high and low regime for each parameter $\{\beta, \kappa, \sigma\}$. The total number of regimes is $2 \times 2 \times 2 = 8$.

$$S_t = \{\beta_H, \beta_L\} \times \{\kappa_H, \kappa_L\} \times \{\sigma_H, \sigma_L\} \tag{A.16}$$

The latent states are estimated by the Hamilton filter given a set of parameters, and the parameter values are estimated to maximize the likelihood of the model. Figure A.2 shows the estimated probabilities of each regime. For the slope parameter of the NKPC κ, the estimated probabilities of high regime declines sharply in terms of the output gap around 2010. The coefficient of marginal cost is roughly stable over time, though volatile state probabilities indicates that the series include sizable noises. The estimation result delivers other interseting observations including a rise of the volatility σ during the Great Inflation period in 1970s and rises of the coefficient of the forward looking inflation term β around the Volcker period in the early 1980s and after the Great Recession. These observations are subject to future research, although I focus on the differences in the differences in the slope parameter in the output gap and the marginal cost representations of the NKPC.

Figure A.2: Smoothed probabilities of high parameter regime



Notes: Smoothed probabilities are the centered 5 quarters average of the state probabilities.

A.2.6 Robustness check (6): dynamic panel estimation using industry level data

One concern regarding the estimation of the NKPC using the aggregate variables is that the estimated marginal cost might include considerable measurement errors. In this subsection, therefore, I investigate the intermediate share in industry level data as an alternative measure of marginal cost for assuring robustness of the analysis. To this end, it is notable that the firm's cost minimization condition can be applied to any factor inputs. Moreover, a number of studies, for example, Basu (1995), Nekarda and Ramey (2013) and Bils et al. (2014), suggest that intermediate inputs are promising in many dimensions. First, adjustment costs for intermediates are considered to be low relative to those for capital or labor. Second, the assumption of no overhead component seems more defensible for intermediates. In addition, using industry level data removes composition bias among industries. I use the KLEMS 2017 dataset to construct the intermediate share. The KLEMS 2017 dataset is annual from 1947 to 2014, covering 65 industries. I focus on 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing.

Since a measure of inflation expectations is not available for each industry, I rely on the rational expectation assumption to estimate the industry level NKPC:

$$\pi_{i,t} = \alpha_i + \beta\pi_{i,t+1} + \gamma\pi_{i,t-1} + \kappa x_{i,t} + e_{i,t+1} \tag{A.17}$$

$$where\ \mathbb{E}_t[\pi_{i,t+1}] = \pi_{i,t+1} + \tilde{e}_{i,t+1} \tag{A.18}$$

with $e_{i,t+1} = \beta\tilde{e}_{i,t+1}$. For varialbe $x_{i,t}$, I consider marginal cost measured by the intermediate share, and the detrended output as a measure of the output gap. α_i is an unobserved industry fixed effect and $e_{i,t+1}$ is the rational expectation error of industry i in period $t+1$. I employ the two step GMM model procedure to correct industry fixed effects, which is called the Arellano-Bond estimator. Since I take the first difference of (A.17) to remove industry fixed effects, valid instruments for moment conditions are one-period more lagged than those in the standard GMM estimator. More discussion is found in Arellano and Bond (1991). The estimation result is presented in Table A.4. The coefficient of the detrended output is insignificant in each specification, which is in line with our GMM estimation with aggregate data. On the other hand, the coefficient of intermediate share is significantly positive and the interaction term is not significant in each case.The purely forward looking NKPC yields similar results to the hybrid NKPC. These results confirm the observations of the baseline estimation that the decline of the slope of the NKPC is not observed in terms of marginal cost.

Table A.4: Dynamic panel GMM estimation of the NKPC in industry level data

	Dependent : Output price inflation π					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.358^{+++}	0.239^{+++}	0.240^{+++}	0.403^{+++}	0.263^{***}	0.264^{***}
	(0.045)	(0.069)	(0.070)	(0.075)	(0.093)	(0.096)
π_{t-1}	0.321^{+++}	0.227^{+++}	0.235^{+++}			
	(0.033)	(0.061)	(0.059)			
$Output_t$	0.0139	0.00224	0.0117	-0.0214	-0.0130	-0.0120
	(0.024)	(0.024)	(0.027)	(0.029)	(0.027)	(0.031)
$postGR_t \times Output_t$			-0.0690			-0.00946
			(0.065)			(0.076)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

	Dependent : Output price inflation π_t					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.359^{+++}	0.244^{+++}	0.244^{+++}	0.393^{+++}	0.261^{***}	0.254^{***}
	(0.045)	(0.072)	(0.073)	(0.073)	(0.093)	(0.095)
π_{t-1}	0.316^{+++}	0.222^{+++}	0.225^{+++}			
	(0.032)	(0.060)	(0.060)			
$IntShare_t$	0.0164^{**}	0.0337^{*}	0.0347^{*}	0.0259^{**}	0.0375^{*}	0.0413^{**}
	(0.008)	(0.018)	(0.020)	(0.013)	(0.019)	(0.021)
$postGR_t \times IntShare_t$			0.00270			0.00799
			(0.005)			(0.005)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: The two step GMM is employed. Output is taken log and detrended by the Hamilton filter. Output price inflation and intermediate price inflation are in log difference while the intermediate share is in log level. Instruments are the second to forth lagged output price inflation rate, the intermediate share, intermediate price inflation, the detrended output. Windmeijer corrected standard errors are reported in parentheses. $AR(p)$ *test* indicates the order of AR process implied by the Arellano-Bond AR(p) test. Industry fixed effects are included. $postGR_t$ is a dummy variable that takes 1 after 2008. A dummy variable for year 2009 is included as an independent variable to control for volatile price inflation of the year, which is not reported in table. Sample period is from 1955 to 2014 for $beforeGR$ and from 1955 to 2014 for $fullsample$, respectively.

B Computation

B.1 Details of equilibrium computation algorithm

ALM. Following the insight of Krusell and Smith (1998), I conjecture that the aggregate law of motion (ALM) Γ is given by a log-linear form:

$$log(\tilde{W}_s) = \ B_{0,s} + B_{1,s} log(\tilde{W}_{-1}) \ , \quad for \ \beta_t = \beta_s \tag{B.19}$$

where s denotes the exogenous state of the economy.

Modified Krusell-Smith algorithm. I sketch the outline of the equilibrium computation. The algorithm takes the following steps in each iteration $m = 1, 2, 3...$

1. Each agent uses the ALM $B^{(m)} = \{B_{0,s}^{(m)}, B_{1,s}^{(m)}\}_{s=1}^{S}$ to forecast the aggregate state variable $\tilde{W}$.

2. Given the aggregate state variables $\{\beta, \tilde{W}\}$, the policy function for aggregate jump variables $f^{(m)}$ is obtained by solving a 3-equation New Keynesian system, i.e., the Euler equation, the NKPC, and the Taylor rule, together with the production function and the market clearing condition. A policy function iteration is used for this procedure.

3. Given the aggregate policy function $f^{(m)}$, households solve their wage setting problem to derive their policy function $h^{(m)}$. A value function iteration is used for this procedure.

4. Given the aggregate policy function $f^{(m)}$ and the individual policy function $h^{(m)}$, I simulate the model economy for T periods and discard the initial T_0 periods to obtain the series of aggregate variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$. I set $T = 51,000$ and $T_0 = 1,000$.

5. Using the simulated variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$, I obtain the suggested ALM $\hat{B}$ by running the OLS of the ALM (B.19). Then, I update the coefficients $B^{(m+1)}$ according to:

$$B^{(m+1)} = \lambda \hat{B} + (1 - \lambda) B^{(m)} \tag{B.20}$$

   where λ is the weight for updating. λ is set to be 0.2.

6. Repeat from step 1 to step 5 until a criteria for convergence of B is attained.

Discretization. I discretize the AR(1) process of exogenous variables using the Rouwenhorst (1995) method. Though the Tauchen (1986) method is widely used in the literature for this purpose, as Kopecky and Suen (2010) pointed out, the Rouwenhorst (1995) method can precisely match several moments of stationary AR(1) process including the first-order autocorrelation and the unconditional variance, even when the process is highly persistent and the number of discretized states is relatively small. State space of endogenous state variables are discretized to use a value function iteration and a policy function iteration, and the linear interpolation is employed to approximate the variables between grids when simulating the economy.

Accuracy check of the ALM. For checking accuracy of the ALM, the Den Haan (2010) statistics is employed. The statistics measures the maximum distance between the aggregate state variables computed according to the ALM $(\tilde{W})_t^{alm}$, and those derived from equilibrium conditions in the simulation $(\tilde{W})_t^{sim}$:

$$DH(B) = sup_{t\in[T_0+1,T]} \; |log(\tilde{W})_t^{sim} - log(\tilde{W})_t^{alm}| \tag{B.21}$$

The critical value is set at $DH(B) = 10^{-3}$, which means that the cumulative error of agents' prediction is smaller than 0.1% over 50,000 periods. The criteria is much more strict than R^2, because R^2 measures the average error in the one-period ahead forecast.

B.2 Comparison with other computation methods

The model developed in this paper is classified into a heterogeneous agent model with aggregate uncertainty, which starts from Krusell and Smith (1998). On the other hand, recent studies propose other computation methods to deal with a heterogeneous agent model. For instance, Reiter (2009) approximates cross-sectional distribution with finite dimensional histograms, whereas Winberry (2016) propose a method to parameterize the distribution by using a family of polynomial functions. Moreover, Ahn et al. (2017) and Kaplan et al. (2016) build a continuous time model where the evolution of the distribution is formulated in the Kolmogorov forward equation and its boundary conditions. These studies use the first order approximation around the stationary distribution in terms of aggregate dynamics to gain the efficiency of computation. However, as discussed in Ahn et al. (2017), this class of solution method cannot capture the sign- and the size-dependency of the effects of an aggregate shock as long as the aggregate dynamics is approximated in the first order. In contrast, I find that the sign- and the size dependency are crucial to accounting for the missing deflation because a large and negative shock such as the Great Recession changes the cross-sectional wage distribution severely. Moreover, the endogenous shifts of the cross-sectional distribution after the initial shocks allow the model to address the excessive disinflation in the subsequent periods. In addition, the ZLB is another reason for us to use a global solution method in terms of the aggregate dynamics, because a local method cannot be used due to the kink of the monetary policy rule.

B.3 Construction of generalized impulse responses

Definition. Following Koop et al. (1996), I define the generalized impulse responses (GIR) as follows:

$$GIR(y, t, \epsilon_0) = \mathbb{E}[y_t|\epsilon_0] - \mathbb{E}[y_t] \tag{B.22}$$

$$= \mathbb{E}\Big[\mathbb{E}[y_t|\beta_0 = \tilde{\beta}_0 e^{-\epsilon_0}, \beta_1 = \tilde{\beta}_1, \cdots, \beta_t = \tilde{\beta}_t, \omega_0 = \tilde{\omega}_0]|\epsilon_0\Big]$$
$$- \mathbb{E}\Big[\mathbb{E}[y_t|\beta_0 = \tilde{\beta}_0, \beta_1 = \tilde{\tilde{\beta}}_1, \cdots, \beta_t = \tilde{\tilde{\beta}}_t, \omega_0 = \tilde{\omega}_0]\Big] \tag{B.23}$$

where y_t, ω_t, and ϵ_t are target variables, state variables, and exogenous shocks, respectively.

Computation. Since the GIR do not have a closed form solution, a simulation based method is employed. The construction of the GIR takes the following steps:

1. Draw an initial state $\tilde{\omega}_0$ and $\tilde{\beta}_0$ randomly.
2. Draw a series of exogenous shocks $\{\tilde{\beta}_s\}_{s=1}^t$ and $\{\tilde{\tilde{\beta}}_s\}_{s=1}^t$ with or without the initial shock ϵ_0, given the initial state.
3. Simulate the economy along with the path of exogenous variables.
4. Repeat the procedure 1-3 for 10,000 times and take the mean to compute expectation.

B.4 Definition of stationary equilibrium

Definition. *A stationary equilibrium is a household's policy function for individual real wages $\tilde{w} = h(\tilde{w}_{-1}, \chi)$, aggregate variables $X = \{\tilde{W}, C, Y, H, \Pi, R\}$, and a probability distribution $p(\tilde{w}, \chi)$, such that*
(i) a household's policy function h solves a recursive wage setting problem,

$$V^{dnwr}\Big(\tilde{w}_{-1}, \chi\Big) = \max_{\tilde{w}} : \ -\frac{1}{1+\eta} e^{\chi} h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\Big(\tilde{w}, \chi'\Big)\Big] \tag{B.24}$$
$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$
$$\tilde{w} \geq \tilde{w}_{-1}/\Pi$$

$$V^{no}\Big(\chi\Big) = \max_{\tilde{w}} : \ -\frac{1}{1+\eta} e^{\chi} h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\Big(\tilde{w}, \chi'\Big)\Big] \tag{B.25}$$
$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$

$$\textit{where} \quad \mathbb{E}\Big[V\Big(\tilde{w}, \chi'\Big)\Big] = (1-\alpha)\mathbb{E}\Big[V^{dnwr}\Big(\tilde{w}, \chi'\Big)\Big] + \alpha\mathbb{E}\Big[V^{no}\Big(\chi'\Big)\Big]$$

(ii) aggregate jump variables X solve the Euler equation (22), the NKPC (36), the monetary policy rule (38), the production function (37), and the market clearing conditions (41), that is,

$$\tilde{W} = Z \ , \quad Y = ZH = C \ , \quad \Pi = \Pi^* \ , \quad R = \Pi^*/\bar{\beta} \tag{B.26}$$

(iii) a probability distribution p is a stationary distribution, that is,

$$p(\tilde{w}',\chi') = \int_{\chi}\int_{\tilde{w}:\tilde{w}'=h(\tilde{w},\chi')} p(\tilde{w},\chi)P(\chi'|\chi)d\tilde{w}d\chi \tag{B.27}$$

(iv) the aggregate hours H satisfies the market clearing condition,

$$H = \left(\int_{\chi}\int_{\tilde{w}} \left\{(\tilde{w}/\tilde{W})^{-\theta_w} Hp(\tilde{w},\chi)\right\}^{\frac{\theta_w-1}{\theta_w}} d\tilde{w}d\chi\right)^{\frac{\theta_w}{\theta_w-1}} \tag{B.28}$$

B.5 Derivation of the second order approximation of the social welfare

The approach adopted in this subsection largely follows Rotmberg and Woodford (1997) and Erceg et al. (2000). I take the second order Taylor expansion of the current social welfare around the deterministic steady state under flexible prices and wages:

$$SW_t \equiv \frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\eta}\int_0^1 e^{\chi_t(j)}h_t(j)^{1+\eta}dj \tag{B.29}$$

$$\begin{aligned}
&\approx \bar{SW} + \bar{C}^{-\sigma}(C_t-\bar{C}) - \frac{1}{2}\sigma\bar{C}^{-\sigma-1}(C_t-\bar{C})^2 \\
&\quad - \bar{h}^{\eta}\int_0^1 (h_t(j)-\bar{h})dj - \frac{1}{2}\eta\bar{h}^{\eta-1}\int_0^1 (h_t(j)-\bar{h})^2 dj \\
&\quad - \frac{1}{1+\eta}\bar{h}^{1+\eta}\int_0^1 \chi_t(j)dj - \frac{1}{2}\frac{1}{1+\eta}\bar{h}^{1+\eta}\int_0^1 (\chi_t(j))^2 dj \\
&\quad - \bar{h}^{\eta}\int_0^1 \chi_t(j)(h_t(j)-\bar{h})dj
\end{aligned} \tag{B.30}$$

$$\begin{aligned}
&= (const.) + \bar{C}^{-\sigma}(C_t-\bar{C}) - \frac{1}{2}\sigma\bar{C}^{-\sigma-1}(C_t-\bar{C})^2 \\
&\quad - \bar{h}^{\eta}\mathbb{E}_j[h_t(j)] - \frac{1}{2}\eta\bar{h}^{\eta-1}\mathbb{V}ar_j[h_t(j)] - \bar{h}^{\eta}\mathbb{C}ov_j[\chi_t(j),h_t(j)]
\end{aligned} \tag{B.31}$$

where $\bar{x}$ is the variable x in the deterministic steady state. From the aggregation of labor service of each worker j,

$$\begin{aligned}
H_t &= \left(\int_0^1 h_t(j)^{\frac{\theta_w-1}{\theta_w}}dj\right)^{\frac{\theta_w}{\theta_w-1}} \\
&\approx \mathbb{E}_j[h_t(j)] - \frac{1}{2}\frac{1}{\theta_w}\frac{1}{\bar{h}}\mathbb{V}ar_j[h_t(j)]
\end{aligned} \tag{B.32}$$

On the other hand, the production function and the resource constraint lead to,

$$\begin{aligned}
H_t = Y_t &= C_t + \frac{\phi}{2}(\Pi_t-\Pi^*)^2 C_t \\
&\approx C_t + \frac{\phi}{2}\bar{C}(\Pi_t-\bar{\Pi})^2
\end{aligned} \tag{B.33}$$

From the individual labor demand,

$$h_t(j) - \bar{h} \approx -\theta_w \bar{h}\big(log(w_t(j)) - log(W_t)\big) + H_t - \bar{H}$$

Taking moments the equation, we obtain

$$\mathbb{V}ar_j[h_t(j)] = \theta_w^2 \bar{h}^2 \mathbb{V}ar_j[log(w_t(j))] \tag{B.34}$$

$$\mathbb{C}ov_j[\chi_t(j), h_t(j)] = -\theta_w \bar{h} \mathbb{C}ov_j[\chi_t(j), log(w_t(j))] \tag{B.35}$$

I assume $\bar{\Pi} = \Pi^*$ and $Z_t = 1$. Then, $\bar{C} = \bar{Y} = \bar{H}$ holds by the resource constraint. By substituting (B.32)-(B.35) into (B.31), I finally obtain (C.38).

C Additional Results

C.1 Unconditional moments

Table C.5 compares the unconditional moments of the model with the data. I simulate the model for 51,000 periods and discarded the initial 1,000 observations to calculate moments. The table also reports the moments in a model without DNWR, which coincides with a 3-equation New Keynesian model. The baseline model with DNWR does fairly well in matching the time-series moments of the data in a number of dimensions including: (1) low standard deviation of price inflation and wage growth relative to that of output and hours, (2) positive skewness of price inflation, wage growth, and real wage, (3) negative skewness of output and hours. On the other hand, the model without DNWR generates positive skewness of output and hours due to the concavity of utility function, which is inconsistent with data.

Table C.5: Unconditional moments of data and model

	Data		Model	
	55Q1-07Q4	87Q4-07Q4	baseline	w/o DNWR
$\sigma(Y)$	1.55	1.09	1.55	0.44
$\sigma(H)$	1.83	1.74	1.55	0.44
$\sigma(\pi^p)$	0.58	0.23	0.60	1.40
$\sigma(\pi^w)$	0.75	0.76	0.66	1.62
$\sigma(W/P)$	0.85	1.16	0.46	0.99
$\sigma(i)$	0.82	0.54	1.26	2.23
$\rho(Y)$	0.85	0.88	0.85	0.86
$\rho(H)$	0.90	0.93	0.85	0.86
$\rho(\pi^p)$	0.86	0.60	0.90	0.87
$\rho(\pi^w)$	0.40	0.09	0.81	0.64
$\rho(W/P)$	0.77	0.03	0.94	0.86
$\rho(i)$	0.95	0.97	0.89	0.87
$Sk(Y)$	-0.49 (-0.05)	0.03 (0.04)	-0.38 (-0.11)	0.34 (0.06)
$Sk(H)$	-0.27 (-0.02)	-0.01 (-0.15)	-0.38 (-0.11)	0.34 (0.06)
$Sk(\pi^p)$	1.25 (0.30)	0.49 (0.17)	1.50 (0.36)	-0.07 (-0.01)
$Sk(\pi^w)$	0.25 (0.00)	0.54 (0.16)	1.75 (0.38)	0.04 (-0.01)
$Sk(W/P)$	0.44 (0.02)	0.45 (0.02)	2.26 (-0.11)	0.36 (0.07)
$Sk(i)$	1.17 (0.15)	0.06 (-0.20)	1.01 (0.23)	-0.02 (-0.00)

Notes: The standard deviation σ, the first order autocorrelation ρ, and the skewness Sk are reported. For the skewness, as well as the standard definition Sk_1, an alternative skewness measure Sk_2 is reported in parentheses. Sk_2 is defined as $Sk_2 = (\mu - Q)/\sigma$ with the mean μ and the median Q, and bounded between -1 and 1. Kim and White (2003) argue that Sk_2 is robust to outliers. Regarding the moments of data, Y is the real GDP, H is the total hours in the non-farm business sector, π^p is the GDP implicit price deflator, π^w is the compensation per hour in the non-farm business sector, and i is the effective federal funds rate. Y and H are taken log and detrended by the HP-filter. π^p and π^w are the quarter-on-quarter growth rate. i is the annual rate divided by 4 (quarterly rate). Sample period is from 1955Q1 to 2007Q4. The end of the sample is determined to exclude the ZLB periods. For computing the moments of the models, we simulate the economy for 51,000 periods and discard the initial 1,000 observations. The model without DNWR is solved by a policy function iteration and simulated under the same parameter values as the baseline model.

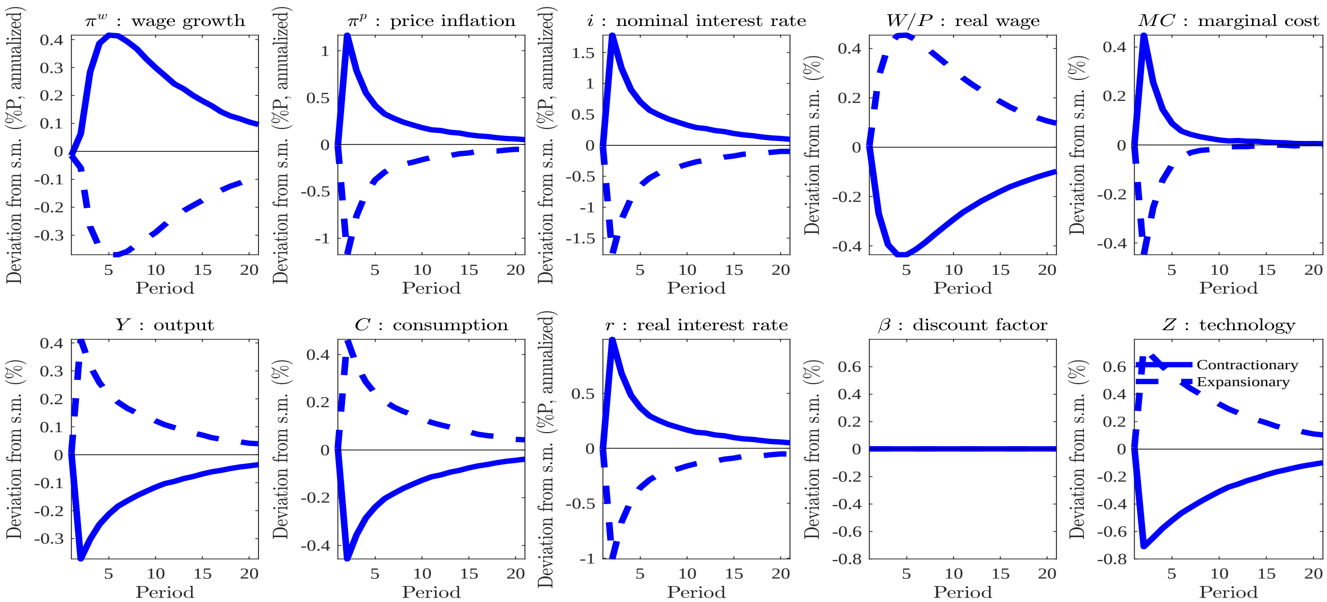
C.2 Effects of supply side shock

This subsection investigates the effects of a technology shock in the model. For this exercise, I consider that the aggregate technology Z_t follows an AR(1) process while keeping the discount factor constant:

$$log(Z_t) = \rho_z log(Z_{t-1}) + \epsilon_{z,t} \ , \ \epsilon_{z,t} \sim N(0, \sigma_z^2) \tag{C.36}$$

For parameterization, I follow Fernández-Villaverde et al. (2015) to set $\rho_z = 0.900$ and $\sigma_z = 0.0025$. Figure C.3 shows the GIR to a 2 S.D. technology shock. Interestingly, a technology shock does not generate significant asymmetry in the GIR. In addition, the relative response of price inflation to that of output is much larger than the response to a demand shock in Figure 4. It might be because a technology shock directly affects firms' marginal cost through Equation (32) and that results in an almost symmetric and large effect on price inflation through the NKPC. This mechanism is considered to be particularly strong given the relatively low degree of price stickiness in my calibration. In literature, on the other hand, Altig et al. (2011) find moderate responses of price inflation to a neutral technology shock in the VAR analysis. They propose to take into account the firm specific capital to match the VAR responses under the micro founded degree of price stickiness. These ingredients might be a potential extension of the model.

Figure C.3: GIR to a 2 S.D. technology shock



Notes: Each series is the deviation from the stochastic mean (s.m.).

C.3 Welfare cost of business cycles

This subsection briefly discusses the welfare implications of DNWR and the ZLB. I define the social welfare as the unconditional expectation of average household utility:

$$SW \equiv \mathbb{E}\left[\frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\eta}\int_0^1 e^{\chi_t(j)}h_t(j)^{1+\eta}dj\right] \tag{C.37}$$

To obtain the intuition behind the welfare losses, I derive the second order approximation of the current social welfare around the deterministic steady state under flexible prices and wages. The derivation is provided in Appendix B.

$$
\begin{aligned}
SW - SW^f \approx & \underbrace{+(1-\bar{C}^{\eta+\sigma})\bar{C}^{-\sigma}\left(\mathbb{E}\big[C_t - C_t^f\big]\right)}_{Aggregate\ mean} \\
& \underbrace{-\frac{1}{2}\sigma\bar{C}^{-\sigma-1}\left(\mathbb{E}\big[(C_t-\bar{C})^2\big] - \mathbb{E}\big[(C_t^f-\bar{C})^2\big]\right) - \frac{\phi_p}{2}\bar{C}^{1+\eta}\mathbb{E}\big[(\Pi_t^p - \Pi^*)^2\big]}_{Aggregate\ variance} \\
& \underbrace{\begin{aligned}& -\frac{1}{2}\theta_w\left(1+\theta_w\eta\right)\bar{C}^{1+\eta}\mathbb{E}\Big[\mathbb{V}ar_j\big[log(w_t(j))\big] - \mathbb{V}ar_j\big[log(w_t^f(j))\big]\Big] \\ & +\theta_w\bar{C}^{1+\eta}\mathbb{E}\Big[\mathbb{C}ov_j\big[\chi_t(j), log(w_t(j))\big] - \mathbb{C}ov_j\big[\chi_t(j), log(w_t^f(j))\big]\Big]\end{aligned}}_{Cross-sectional\ dispersion}
\end{aligned} \tag{C.38}
$$

where $Var_j(\cdot)$ and $Cov_j(\cdot,\cdot)$ are the cross-sectional variance and covariance, respectively. SW^f denotes the social welfare under flexible prices and wages. Equation (C.38) gives three sources of the welfare losses in this economy: (1) the mean of consumption, (2) the variance of consumption and price inflation, and (3) the cross-sectional inefficient wage dispersion. It is worth pointing that the cross-sectional wage dispersion cannot be summarized by aggregate variables while Erceg et al. (2000) demonstrate that, in the Calvo-type staggered wage model without idiosyncratic shocks, the dispersion is represented by the variance of the aggregate wage growth rate.

Since the social welfare of the economy does not have the exact closed form solution, I numerically compute it by simulating the economy with a large number of households for a long period of time and taking the unconditional mean. Table C.6 reports the social welfare and its relevant measures. The first thing to note is that DNWR generates sizable welfare losses in the stationary equilibrium in column 1 by generating the cross-sectional wage dispersion that results in the inefficiency of production through the Jensen's inequality. Once dynamics are considered, moreover, the consumption equivalent losses is enlarged to 2.77% in column 2. The decrease of the social welfare compared to the stationary equilibrium arises from the time-series variations in consumption and price inflation, and the larger cross-sectional wage dispersion. Though the mean consumption is higher than the stationary equilibrium due to the precautionary savings, the benefit of it is overwhelmed by the adverse factors. The presence of the ZLB further deteriorates the social welfare under the standard Taylor rule in column 3 by amplifying the variance of the aggregate consumption and the cross-sectional dispersion. The contractionary bias of the Taylor rule under the ZLB suggested by Basu and Bundick (2017) decreases the mean consumption as well. On the other hand, the history dependent rule in column 4 offsets the adverse effects of the ZLB by committing the future low interest rate policy once constrained at the ZLB. Interestingly, the welfare under the history dependent rule is slightly higher than the Taylor rule without the ZLB.

Overall, the analysis in this subsection uncovers that the model with DNWR together with the ZLB generates substantial welfare losses in both stationary and dynamic settings. The results are in sharp contrast to the previous studies that detect the moderate welfare effects of DNWR. For example, Kim and Ruge-Murcia (2009) find that the optimal inflation rate to offset the welfare cost of DNWR is small. Moreover, Coibion et al. (2012) claimed that DNWR improves the social welfare under the ZLB by reducing the inflation variations and therefore reducing the likelihood of the ZLB binding. One potential cause behind the differences would be the presence of heterogeneity, but further investigation is left for future research.

Table C.6: Welfare measures

	(1)	(2)	(3)	(4)
	Stationary		Dynamics	
	Equilibrium	Taylor rule without ZLB	Taylor rule with ZLB	History dependent rule with ZLB
Social welfare:				
C.E.(%)	-2.27	-2.77	-3.36	-2.64
$Mean(C)$	0.9947	0.9982	0.9938	0.9975
$Std.(C)$ (%)	0.00	1.55	2.26	1.67
$Mean(\pi^p)$	0.50	0.51	0.30	0.55
$Std.(\pi^p)$ (%)	0.00	0.66	0.48	0.57
$Std._j(log(w))$ (%)	5.72	7.04	10.20	6.56
$Corr._j(log(w), \chi)$	0.44	0.34	0.16	0.38

Notes: $C.E.$ denotes the consumption equivalent welfare losses compared from the economy with flexible price and wage. $Std._j(\cdot)$ and $Corr._j(\cdot,\cdot)$ are the cross-sectional standard deviation and correlation, respectively.

D Model Comparison

D.1 Flexible wage model

Wage setting. The friction less labor market equilibrium implies that real wage is equalized to the marginal rate of substitution.

$$\frac{W_t}{P_t} = \frac{H_t^{\eta}}{C_t^{-\sigma}} \tag{D.39}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{D.40}$$

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \frac{\theta_p}{\phi_p} \left(\frac{W_t}{P_t} - 1 \right) \tag{D.41}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{D.42}$$

$$Y_t = H_t = C_t + \frac{\phi_p}{2} (\Pi_t - \Pi^*)^2 C_t \tag{D.43}$$

Computation. I discretize the state space and use a policy function iteration to derive a global solution.

D.2 Quadratic wage adjustment cost model

Wage setting. Households are subject to the following budget constraint with quadratic wage adjustment cost:

$$C_t + \frac{A_t}{P_t} \leq (1 + \tau_w) \frac{W_t}{P_t} H_t - \frac{\phi_w}{2} (\Pi_t^w - \Pi^*)^2 H_t + R_{t-1} \frac{A_{t-1}}{P_t} + \frac{\Phi_t}{P_t} \tag{D.44}$$

where the notation follows the benchmark model. The FOC yields the wage Phillips curve:

$$(\Pi_t^w - \Pi^*)\Pi_t^w = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{H_{t+1}}{H_t} \right) (\Pi_{t+1}^w - \Pi^*)\Pi_{t+1}^w \right] + \frac{\theta_w}{\phi_w} \left(\frac{H_t^\eta}{C_t^{-\sigma}} - \frac{W_t}{P_t} \right) \tag{D.45}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{D.46}$$

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \frac{\theta_p}{\phi_p} \left(\frac{W_t}{P_t} - 1 \right) \tag{D.47}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{D.48}$$

$$Y_t = H_t = C_t + \frac{\phi_p}{2} (\Pi_t - \Pi^*)^2 C_t + \frac{\phi_w}{2} (\Pi_t^w - \Pi^*)^2 C_t \tag{D.49}$$

Computation. The same solution method is used as the flexible wage model.

Calibration. I calibrate the parameter value for the degree of wage stickiness ϕ_w according to the micro evidence reported by Barattieri et al. (2014). They identify the frequency of individual wage changes to be 23.9% per quarter as the midpoint of the estimates under different plausible assumptions. The estimate implies the slope of the wage Phillips curve to be 0.076.

D.3 Asymmetric wage and price adjustment cost model

Model equations. The model consists of the Euler equation, the price Phillips curve, the wage Phillips curve, the Taylor rule, and the resource constraint. Derivations follow Aruoba et al. (2017).

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{D.50}$$

$$\Phi'_p(\Pi^p_t)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) \Phi'_p(\Pi^p_{t+1})\Pi_{t+1} \right] + \theta_p \left(\frac{W_t}{P_t} - 1 \right) \tag{D.51}$$

$$where\ \Phi_p \equiv \phi_p \left(\frac{exp(-\psi_p(\Pi^p_t - \Pi^*)) + \psi_p(\Pi^p_t - \Pi^*) - 1}{\psi_p^2} \right)$$

$$\Phi'_w(\Pi^w_t)\Pi^w_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{H_{t+1}}{H_t} \right) \Phi'_w(\Pi^w_{t+1})\Pi^w_{t+1} \right] + \theta_w \left(\frac{H^\eta_t}{C_t^{-\sigma}} - \frac{W_t}{P_t} \right) \tag{D.52}$$

$$where\ \Phi_w \equiv \phi_w \left(\frac{exp(-\psi_w(\Pi^w_t - \Pi^*)) + \psi_w(\Pi^w_t - \Pi^*) - 1}{\psi_w^2} \right)$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{D.53}$$

$$Y_t = H_t = C_t + \Phi_p(\Pi^p_t)C_t + \Phi_w(\Pi^w_t)C_t \tag{D.54}$$

where ϕ and ψ govern the slope and the curvature of the Phillips curve, respectively.

Computation. Following Aruoba et al. (2017), I use the second order perturbation method around the steady state to solve the model.

Calibration. The parameter values for adjustment cost functions are set according to the posterior mean of Aruoba et al. (2017) for the sample of 1960Q1-2007Q4. Other parameters are identical to the baseline model.

Table D.7: Calibrated parameter values in an asymmetric wage and price adjustment cost model

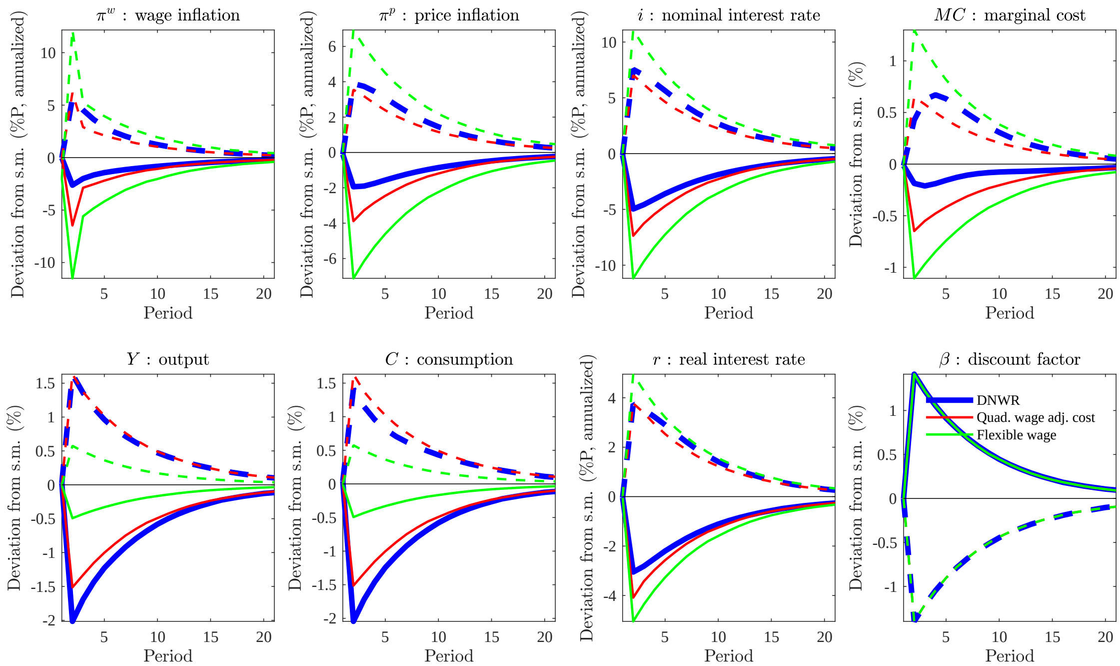
ϕ_p (ξ_p)	ψ_p	ϕ_w (ξ_w)	ψ_w
450 (0.87)	150	*28.1 (0.57)*	67.4

Notes: ξ is the Calvo parameter corresponding to ϕ with $\theta/\phi = (1-\xi)(1-\beta\xi)/\xi$. Italic values are the author's calculation based on Aruoba et al. (2017). For example, the posterior mean of Aruoba et al. (2017) for the slope of price Phillips curve is 0.02. In our model, $0.02 = \theta_p/\phi_p$ and $\theta_p = 9$ implies $\phi_p = 450$.

D.4 Comparison of GIR

Figure D.4 presents the GIR to discount factor shocks in the flexible wage model and the quadratic wage adjustment cost model as well as our baseline model with DNWR.

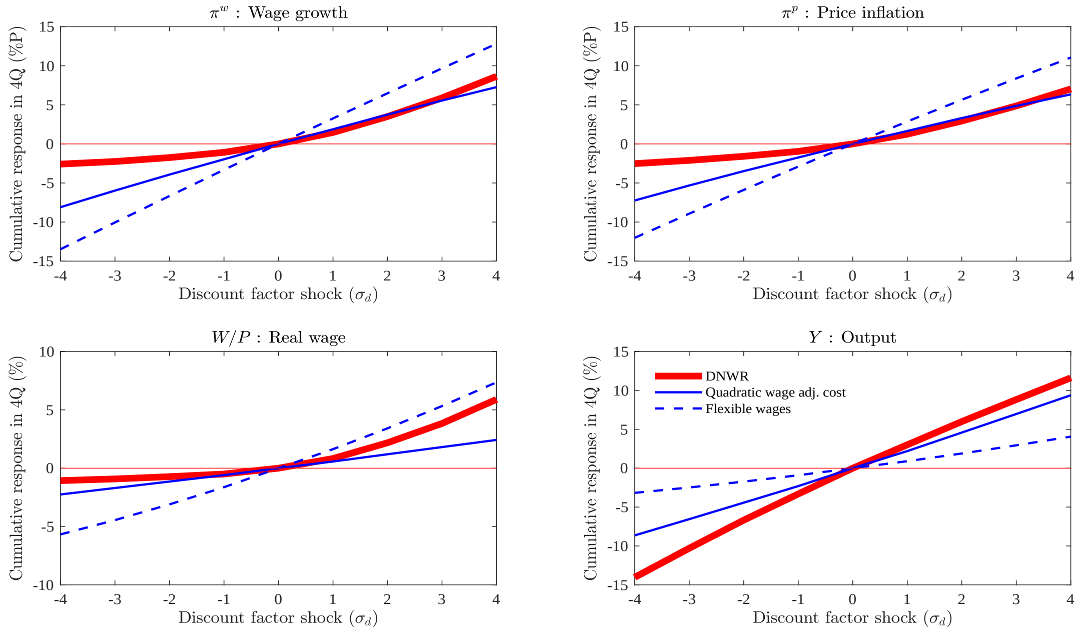
Figure D.4: GIR in different models



D.5 Comparison of non-linearity

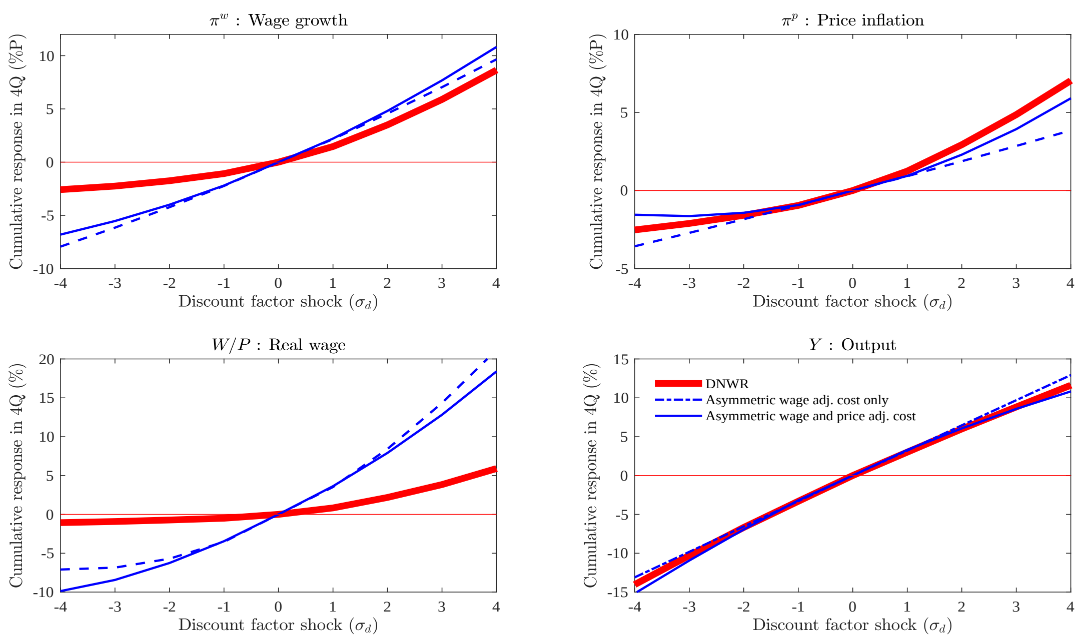
To compare the non-linearity in terms of the responses to exogenous shocks in each model, Figure D.5 and Figure D.6 display the cumulative responses of the selected aggregate variables in the initial 4 quarters after different size and direction of discount factor shocks.

Figure D.5: Comparison of non-linearity (1)



Notes: Each panel displays the cumulative responses in the initial 4 quarters after discount factor shocks.

Figure D.6: Comparison of non-linearity (2)



Notes: Each panel displays the cumulative responses in the initial 4 quarters after discount factor shocks. For the asymmetric wage adjustment cost, the parameter for the asymmetry of price adjustment cost ψ_p is set at zero.