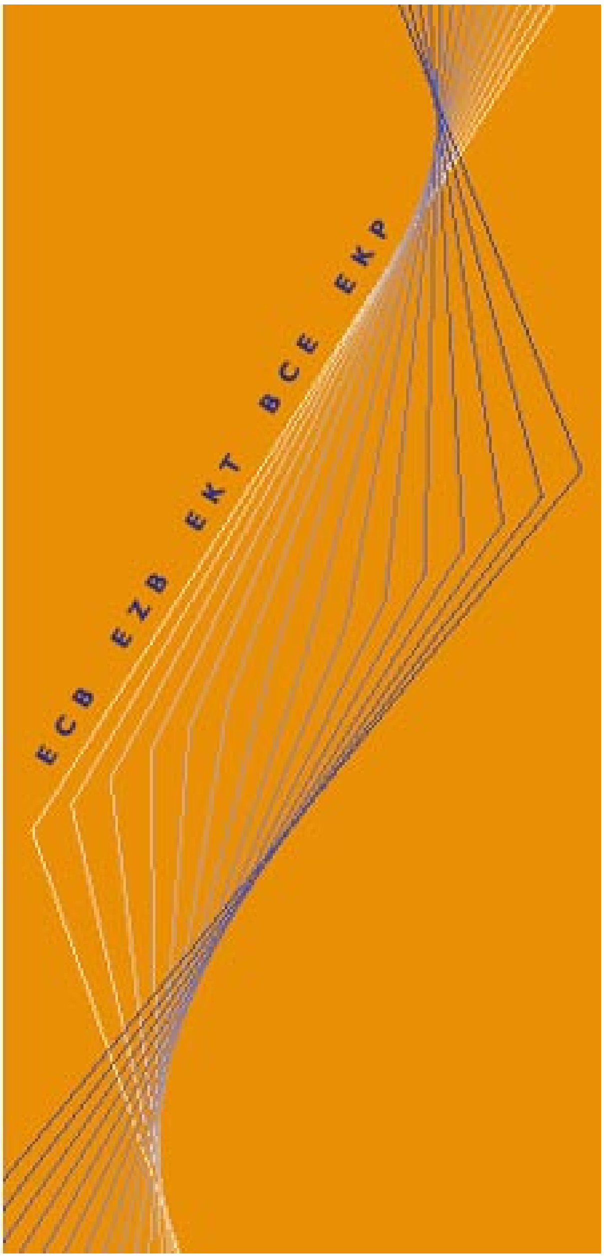
EUROPEAN CENTRAL BANK

WORKING PAPER SERIES



WORKING PAPER NO. 17

ALTERNATIVE MEASURES OF THE NAIRU IN THE EURO AREA: ESTIMATES AND ASSESSMENT

BY SILVIA FABIANI AND RICARDO MESTRE

MARCH 2000

EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

WORKING PAPER NO. 17

ALTERNATIVE MEASURES OF THE NAIRU IN THE EURO AREA: ESTIMATES AND ASSESSMENT

BY SILVIA FABIANI AND RICARDO MESTRE*

MARCH 2000

* Both authors are economists at the European Central Bank, Directorate General Research, Econometric Modelling Division. The opinions expressed in this work do not necessarily reflect the views of the European Central Bank. The authors are grateful to Gabriel Fagan, to colleagues of the European Central Bank and to an anonymous referee for helpful comments. Errors remaining in the text are the sole responsibility of the authors.

Address	**Kaiserstrasse 29** **D-60311 Frankfurt am Main** **Germany**
Postal address	**Postfach 16 03 19** **D-60066 Frankfurt am Main** **Germany**
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

ISSN 1561-0810

Abstract

The paper focuses on the measurement of the NAIRU (Non-Accelerating-Inflation-Rate-of-Unemployment) for the euro area and assesses the usefulness of different methodologies developed in the literature to estimate this unobservable variable at the aggregate level.

After reviewing the theoretical framework underlying the most common estimation approaches, it presents several estimates of the area-wide NAIRU based on a number of direct (or statistical) techniques. The latter range from simple univariate filtering approaches to more complex multivariate methods based on Phillips curve relationships. The different estimates of the aggregate NAIRU appear to be consistent and robust with respect to alternative specifications, methodologies and choice of the inflation indicator. They also show significant inflation forecasting ability and are able to produce sensible measures of the output gap, therefore providing some ground to argue that unemployment and the unemployment gap may be a useful variable to analyse short-term economic developments at the euro area level.

JEL Classification System: E24, E31, C14, C22.

Keywords: NAIRU, Phillips curve, Kalman filter.

1 Introduction

This paper presents a preliminary attempt at measuring the NAIRU (Non-Accelerating-Inflation-Rate-of-Unemployment) in the euro area and assessing the usefulness of different methodologies developed in the literature to estimate this unobservable variable at the whole area level. In the context of the models analysed, the gap between current unemployment and the NAIRU summarises the extent to which inflationary and disinflationary pressures exist in the labour market. A reliable measure of the area wide NAIRU may hence potentially serve as an indicator, among many others, for evaluating the outlook for future inflation developments in the economy.

The NAIRU is often identified in the literature with the concept of "natural" or structural unemployment, that is, the component of the unemployment rate that is ultimately pinned down by the structural, institutional and behavioural characteristics of the economy and does not depend on cyclical factors. However, although the two notions might be difficult to disentangle empirically, they need not necessarily coincide in the short run. The unemployment rate consistent with stable inflation might in fact deviate temporarily from its long-run structural level, especially when shocks have highly persistent effects on the labour market.[1]

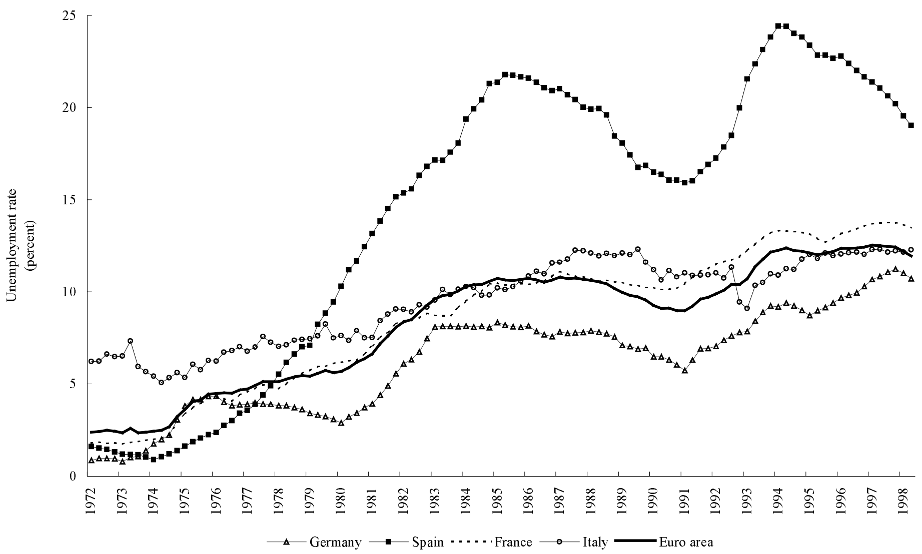
Drawing on this conceptual distinction, this paper does not attempt to investigate, at the aggregate level, the structural factors underlying and driving the performance of the euro area labour market. This would be a daunting task, given the cross-country differences in the institutional characteristics, the specificity of the shocks that affect them and the general unavailability of structural information at the euro area level. Rather, this work focuses explicitly on the methodological issue of the potential usefulness of aggregate NAIRU measures obtained with different empirical techniques based on time series analysis. In particular, the techniques considered here are those which are often labelled in the literature as "direct" methods for the computation of the NAIRU. Within this framework, the focus is mainly on methods based on the information provided by Phillips curve-type relationships between unemployment and inflation.

A notable feature of this paper is that it adopts an euro area-wide approach embedding the estimation of the NAIRU in a set of equations, which treat the euro area as a single economy. For the "direct" methods considered in this paper, it is essentially an empirical

[1] Actual unemployment could drop below the NAIRU and inflation accelerate even if unemployment remains above its structural value. The slowness of the NAIRU in returning to its structural level is often described in the literature as "speed limit effect". See Estrella and Mishkin (1998).

issue as to whether a country-specific or an area-wide approach would be most useful for estimating inflationary pressures arising from the labour market in the area, since such methods are basically statistical. A priori arguments can be made either way: differences in economic structure and potential non-linearities in national Phillips curves would suggest aggregating country-based estimates. On the other hand, the Phillips curve is certainly a more relevant concept in a large closed economy such as the euro area as a whole. The area-wide approach has a number of advantages including simplicity, directly addressing the issue of interest and contributing to focusing discussion on euro area, rather than country specific factors. Moreover, it allows overcoming the difficulties of examining and comparing country-specific institutional factors and the problems related to estimating the NAIRU in small open economies. However, it is certainly the case that member countries' unemployment rates differ by a wide margin, as figure 1.1 clearly depicts.[2]

Figure 1.1. Unemployment rates in the Euro area



The figure collects data for the four largest countries of the area, which form a significant share of the area unemployment. The very different unemployment stories that can be told at the country level, and the role that institutional factors may be playing in these stories, suggests that the results of the area-wide approach may be affected by aggregation biases. The extent to which this is the case can only be established by carrying out the analysis both at the country-specific and at the area-wide level and compare the results, a field that

2 The data reported in the figure are from the ECB area-wide model database. For details, see Fagan, Henry and Mestre (2000).

offers scope for further research but will not be pursued in the current paper. In the meantime, it is necessary to be cautious when trying to extract lessons from an aggregate analysis.

The paper is organised as follows. Section 2 briefly recalls the theoretical framework underlying the most commonly adopted approaches to estimate the NAIRU, within the class of “direct” or statistical methods. Section 3 provides an overview and discussion of such modelling approaches. Section 4 derives some preliminary aggregate estimates for the euro area, based both on models in which the NAIRU is treated as a constant and on models in which it is allowed to change over the sample period. The analysis focuses primarily on the robustness and precision of such estimates with respect to alternative model specifications. Section 5 is devoted to the description and empirical implementation of a Kalman filter approach to derive a time varying NAIRU. Section 6 provides an assessment of the empirical relevance of the NAIRU estimates based, on one hand, on the out-of-sample inflation forecasting performance of the unemployment rate and the unemployment gap and, on the other hand, on the use of the derived NAIRU measures for the computation of output gaps. Conclusions are presented in Section 7.

2 Factors playing a role in the determination of the NAIRU

Overall underlying framework

The Phillips curve underlying our attempt at computing an area-wide measure of the NAIRU based on a multivariate framework can be thought of as arising from a wage-price system describing the behaviour of ‘firms’ and ‘workers’. The basic, general properties assumed to hold in the interaction of the two types of agents lead to a set of equations with precisely defined long-run and short-run properties. In the long run, a *nominal static (first-order) homogeneity* condition is assumed to hold, a property by which equilibrium values can be expressed in terms of relative prices. Another assumption is further made, namely *nominal dynamic (second-order) homogeneity*, by which equilibrium conditions are independent from nominal factors and thus do not depend on the final level of inflation. The first condition can be described by (2.1):

$$p_{SS} = \mu_{SS} + w_{SS} - q_{SS} \qquad (2.1)$$

where p_{SS} is the price level, w_{SS} the nominal wage rate, q_{SS} labour productivity and μ_{SS} the mark-up of prices over labour costs. According to this expression, in the steady state (*ss*) the price level p_{SS} is equal to the labour cost w_{SS} - q_{SS} plus a term measuring the gap between the two, hence ensuring that prices and costs grow in steady state at the same pace. The *level* of the resulting real wage depends on labour productivity, assumed to be driven by technology factors, and the particular value taken by μ_{SS}, which is determined by factors not explicitly included in the equation.

Within this context, an assumption maintained throughout this work is that μ_{SS} does not depend on nominal factors, making the system independent from nominal variables in the long run. As a consequence, the steady-state real wage does not depend on nominal factors, although some short-run effects may be present due to, e.g., market imperfections. A consequence of this fact is that inflation and unemployment are only linked in the short term. Another important consequence is that such links can be analysed using different measures of inflation, such as price inflation (using alternative measures of the price level), unit labour costs or nominal wage inflation. In equilibrium all these variables must grow at the same pace and therefore represent interchangeable measures of inflation.[3] In this paper, some empirical exercises will also be based on price-wage equations, i.e. equations implicitly measuring the real wage not corrected for labour augmenting technology growth. Although, as already stated, this relative price will be assumed independent in the long run from nominal factors, price-wage equations may prove a necessary tool for the short-term analysis of the impact of changes in unemployment on general economic conditions.

Dynamic elements

Dynamic components of the above curve (2.1) are an essential element of the analysis, but are less grounded on theoretical terms. Probably the most widespread dynamic conceptual framework is Gordon's *triangle model* (see Gordon, 1982), an application of which is presented below. More elaborate versions of Gordon's approach have been derived, some based on microeconomic foundations (see Goodfriend and King, 1997 for a good overview). The approach initiates in a framework related to (2.1), in which inflation is linked to supply-side factors and some zero-mean measure of demand pressures, as expressed by (2.2). This expression is no longer a representation of the steady state, but a factual representation of the time-path followed by inflation at each point in time. Hence, p_t is the actual price level observed at time *t*, while x_t* is a measure of the steady state in

3 Except nominal wage, which also grows with labour productivity. Precisely because of this feature estimation of Phillips curves based on nominal wages are less common than alternative specifications, notwithstanding the initial thread by A.W. Phillips.

(2.1), and thus (x_t - x_t*) is a measure of deviations from the steady state. (The variable z_t collects additional information the analyst may find relevant, mainly supply-side factors.)

$$A(L)\Delta^2 p_t = \gamma z_t + \Theta(L)\left(x_t - x_t^*\right) + \varepsilon_t \tag{2.2}$$

There is a one-to-one link between (2.1) and (2.2), but its derivation is left for Annex A. Focusing on the expression itself, there are two elements to highlight: the presence of inflation in first differences, and that of a term x measuring deviations from a long-run (starred) value. The first factor (i.e., the presence of $\Delta^2 p_t$) stems from the lack of long-term effects of nominal variables on real ones, which translates into an inflation process that only depends on nominal factors *in the long run*. In particular, inflation expectations are assumed to be based only on nominal variables. This means that equation (2.2), which has only real variables on the right-hand side, cannot be used to pin down these expectations and therefore inflation must enter in difference form.[4] The second factor stems from the presence of long-run equilibrium values (embodied in the generic variable x*) that are independent from nominal factors, as was the case in (2.1). Such long-run values can be pinned down by real unit labour costs, potential output, the NAIRU, or a mix of them. The focus in the present paper will be on the NAIRU (i.e., x^* is in our notation u^*), but the inclusion of other measures will be occasionally necessary.[5]

A related issue is whether to model levels or first differences of inflation. Traditionally—and we will stick to the convention—a Phillips curve is understood to model first differences of the data, while what has been termed in the literature as 'wage curve' models levels. This distinction is up to a point artificial, but has been enshrined very particularly after the publication of Blanchflower and Oswald's book (see Blanchflower and Oswald, 1994; see also Roberts, 1997 and Whelan, 1997 for a discussion on the relevance of the subject). Following Gordon (1998) we will include level terms among the additional components generally labelled in (2.2) as variable z, in the form of real unit labour costs after deducting the mean.

Other variables

Last but not least, other variables may have a bearing on the NAIRU. Most authors have considered what they term as supply-side variables, which may capture shocks of a

4 Obviously, agents might use (2.2) to set more precisely inflation expectations a limited number of periods ahead, but they would look elsewhere to pin down expectations at a longer horizon.

5 More precisely, real unit labour costs are an important element in some of the specifications.

different nature than those normally associated with a Phillips-type relationship. The set of potential candidates for the euro area is relatively scarce: although some factors or events have had an important impact on inflation in some member countries, it is difficult to find one global enough to warrant its inclusion at the area level. As variables potentially important, foreign prices in various forms will be retained, as some forms of taxation (i.e., direct taxes, social security contributions and their sum). The latter have been found to be not relevant on empirical grounds, and they have been dropped from the models presented.

3 "Direct" methods for estimating the NAIRU: methodological background

The NAIRU is not directly observable and therefore it has to be inferred from the analysis of observable variables related to its definition. The empirical methods adopted in the literature to address this problem can be grouped into two broad categories: the structural approach and the one often labelled as "direct". In the former, the NAIRU is computed as the equilibrium outcome of a structural model representing aggregate price and wage behaviour (see Layard et al, 1991; Morgan and Mourougane, 1999).[6] In the latter, NAIRU measures are derived on the basis of the time series analysis of unemployment and, in multivariate contexts, inflation and other relevant variables. Direct methods are generally easier to implement than structural ones, since they do not attempt at detailed specification and analysis of the underlying behaviour of economic agents.[7] They can in principle provide useful tools for deriving predictions of inflationary pressures, developing measures of uncertainty surrounding the NAIRU estimates, investigating the presence of persistence/hysteresis effects in unemployment dynamics, testing whether such effects are symmetrical.

This section focuses on the main methodological features of this approach, with the aim of building a general framework in which to cast the numerous models developed in the literature, underline their advantages, drawbacks and implications.

Univariate methods

Univariate methods focus uniquely on the unemployment rate time series and decompose it into a trend component, identified as the NAIRU, and a residual (cyclical) component.

[6] This type of analysis is not dealt with in the present work and it is left as a subject for later investigation.
[7] This aspect is particularly relevant in contexts where the structural and institutional characteristics of the labour market are not well identifiable, as is the case of the euro area.

The idea behind such a strategy is that unemployment fluctuates around the NAIRU, i.e. there exist balancing forces in the economy that enable the labour market to reach an equilibrium in the long run. The identification of the two components can be based either on filtering techniques, among which the most widely used are the Hodrick-Prescott (HP) and the Baxter-King filters[8], or on statistical methods, such as the unobserved component (UC) model developed by Watson (1986) or the one proposed by Beveridge and Nelson (1981).

Although quite easy to implement and useful for obtaining updated measures, the univariate approach has a few drawbacks. The most relevant is that it is “atheoretical”, in the sense that it leaves the interaction between unemployment and other economic variables completely indeterminate. In particular, it does not take into account inflation dynamics; hence there is no guarantee that the results provide a measure of the underlying NAIRU, which is useful in the context of explaining the behaviour of inflation. Moreover, results are extremely sensitive to arbitrary choices concerning, for example, the smoothing parameter in the case of the HP filter or the restrictions imposed on the time profile of the trend component in the case of the UC model. Last but not least, most of these filters are affected by lack of precision of end-of-sample estimates of the unobserved variable.

Multivariate methods

The information provided by the inflationary process (and possibly also other variables) proves empirically quite helpful in order to get a less arbitrary decomposition of unemployment into the NAIRU and the cyclical component. The natural rate theory postulates a causal relationship between inflation and the tightness of the labour market, which can be expressed as a generalised expectation augmented Phillips curve:

$$\Delta p_t - \Delta p_t^e = C(L)(\Delta p_{t-1} - \Delta p_{t-1}^e) + \Theta(L)(u_t - u_t^*) + \gamma z_t + \varepsilon_t \qquad (3.1)$$

where Δp is the inflation rate, Δp^e expected inflation, u the unemployment rate, u^* the NAIRU, $C(L)$ and $\Theta(L)$ polynomials in the lag operator, z a vector of factors (usually supply shocks) influencing inflation other than the disequilibrium in the labour market and ε is iid with mean zero and variance σ^2_ε. The lagged dependent variable included in the right-hand side of the equation captures that part of inflation dynamics due to inertial effects.

8 See Baxter and King (1995).

The estimation of (3.1) requires a series of inflationary expectations, which is not model-endogenous and hence entails a certain degree of arbitrariness. One of the most common formulations is the random walk model: $\Delta p_t^e = \Delta p_{t-1}$. The underlying idea is that inflation is a highly persistent phenomenon, for which it is generally quite hard to reject the unit root hypothesis. Alternative specifications for expected inflation, however, are found in the literature. They include recursive autoregressive forecasts, obtained from the recursive estimation of a model of the form $\Delta p_t^e = \delta + D(L)\Delta p_{t-1}$, or the use of data from surveys (Staiger et al, 1996; Fabiani et al, 1997).

The backward-looking hypothesis for inflationary expectations allows rewriting the Phillips curve as:

$$A(L)\Delta p_t = \Theta(L)(u_t - u_t^*) + \gamma z_t + \varepsilon_t \qquad (3.2)$$

where the lagged values of the dependent variable capture both inertial effects related to the speed of price adjustment and expectation formation. If $A(1)$ equals unity there is a "natural rate" of unemployment consistent with a non increasing rate of inflation.[9] In that case, a further difference of variable p can be taken in (3.2). Price dynamics depend on both the level and change of the unemployment gap. Level effects are captured by $\Theta(1)$, while change effects by the individual coefficients themselves. The latter are generally found to be quite significant in most empirical implementations of the Phillips curve, providing support to what in the literature is defined as persistence or, as a limit case, "hysteresis" effects. In both cases the pattern of unemployment itself has long lasting effects on the natural rate.[10] However, while persistence is consistent with a long-run equilibrium towards which the NAIRU, affected by shocks, tends to converge more or less slowly, pure hysteresis implies that a long-run value for the NAIRU cannot be identified. In this case the unemployment gap enters the Phillips curve only in first differences and not in levels. In terms of the time series properties of observed unemployment, this entails the presence of a unit root in the process.[11]

[9] Indeed, the acronym NAIRU, for non-accelerating-inflation-rate-of-unemployment, implies too many derivatives. Its correct version would be NIIRU. Clearly, this comment only holds for a specification with no explicit expectations term in z_t.

[10] Most explanations for mechanisms that might induce such effects focus on the behaviour of labour market participants, the changes in their productive capacity caused by unemployment, the matching process between workers and jobs and the resulting consequences for wage bargaining (Lindbeck and Snower, 1988; Blanchard and Summers, 1986).

[11] It is difficult to determine, within the analytical framework of equation (3.2), whether the presence of a unit root in unemployment is due to true hysteresis or to the fact that the equilibrium rate itself is non stationary due to some structural changes. In other words, the conventional definition of hysteresis fails to

Before turning to a description of the various methodologies available in order to estimate the NAIRU on the basis of the Phillips curve, it is worth outlining some general, and often controversial, issues which have been addressed in the literature on the subject.

- The Phillips curve can be specified in terms of wage rather than price dynamics. As Gordon (1998) points out, the (constant or time varying) NAIRU estimates obtained with the two alternative specifications can differ quite significantly from each other.[12] Further, the NAIRU estimates are sensitive to the price indices chosen to measure inflation.[13] This fact, which could be taken as implying the non-existence of the concept, probably originates in the imprecise nature of any estimate of the NAIRU. In this sense, presenting alternative estimates is helpful: either all measures look alike and the robustness of the exercise is enhanced, or they differ significantly and the exercise is flawed.[14]
- The right-hand side variables of the Phillips curve can enter either as lagged or as contemporaneous values. In Staiger et al (1996) the variables included in the vector z are allowed to enter contemporaneously; in Gordon (1996) unemployment enters contemporaneously.
- Different specifications of the Phillips curve lead in general to different point estimates of the level of the NAIRU and different confidence intervals around such estimates. The precision of the NAIRU time series is found to depend on a number of factors, such as the size of the high frequency variation of the NAIRU, the inclusion of a drift in the random walk process representing it and the specific form chosen to model such a drift.
- The choice and specification (e.g. the lag structure) of the variables included in the vector z are relevant for correctly estimating the relationship between unemployment and inflation. If such factors are well specified, the resulting measure of the NAIRU is consistent with stable inflation in their absence. As Gordon (1996) argues, a

take into account the possibility that some large shocks change the parameters of the Phillips curve. Bianchi and Zoega (1998) adopt a Markov switching regression model to identify the dates of infrequent changes in the mean of the unemployment time series and show that in most European countries unemployment persistence is much reduced once the changing mean rate is taken into account. This means that persistently high unemployment is explained not just by slow adjustment towards a constant natural rate but rather by a higher value of the natural rate. The shifting mean value specification for equilibrium unemployment is compatible with models that predict changes in the natural rate driven by structural factors.

12 While Gordon's result is empirically true, it cannot be true in reality. Suppose that the unemployment rate was at the CPI-NAIRU, then CPI inflation would be non-accelerating. However, if this was not equal to PPI-NAIRU then PPI inflation would be accelerating or decelerating indefinitely and the gap between the two indices (and even the two inflation rates) would be exploding.

13 Staiger et al (1996).

14 Alternatively, the NAIRUs for the different price indices could be estimated jointly subject to the equality restriction.

Phillips curve that does not include any variable as a proxy for the influence of supply factors might create an omitted variable problem, producing unreliable predictions.

- The relationship between inflation and the tightness of the labour market might be asymmetrical, in the sense that excess demand conditions might be more inflationary than excess supply conditions are disinflationary (Laxton, Meredith and Rose, 1994; Clark, Laxton and Rose, 1995). This feature would have relevant policy implications: an overheating of the economy would in fact necessitate a more severe tightening in monetary conditions in order to keep price stability under control. Asymmetry, induced by a number of structural factors, might also characterise the presence of hysteresis effects in the labour market.[15]

Multivariate filters

As already mentioned, a limit of one of the most common filtering technique, the HP filter, is that it generates the NAIRU without taking into account information on inflation. A simple way to get around this drawback is to choose the value of the smoothing parameter that minimises the residual of the Phillips curve (3.2). The NAIRU is then identified as the filtered series of unemployment that provides the best statistical fit.

Alternatively, a multivariate extension of the HP filter makes the NAIRU estimate depend both on the smoothing parameter and on the estimated coefficients of the Phillips curve, thus introducing some structural information in the trend-cycle decomposition (Laxton and Tetlow, 1992). The multivariate filter estimates the NAIRU as the series of u^* that simultaneously minimises the squared unemployment gap subject to a smoothness constraint and a goodness of fit restriction from the Phillips curve:

$$\min \sum_{t=1}^{T} (u_t - u_t^*)^2 + \lambda_p [(u_{t+1}^* - u_t^*) - (u_t^* - u_{t-1}^*)]^2 + \lambda_\varepsilon \varepsilon_t^2 \qquad (3.3)$$

where ε_t is the residual from (3.2) and λ_ε is a weight attached to it. The weights on the restrictions are either fixed a priori or, in a more sophisticated approach, jointly estimated with the parameters of the Phillips curve (Cote' and Hostland, 1994).

15 Giorno, Deserres and Sturm (1997) find some (not very conclusive) evidence of asymmetrical hysteresis for France, Italy and Germany.

Constant NAIRU

If the NAIRU is treated as constant over the whole sample period it can be directly computed from the estimated parameters of equation (3.2). Given the constancy of u^*, the term $\Theta(L)(u_{t-1}-u^*)$ in the Phillips curve can in fact be rewritten as $\Theta(L)(u_{t-1})-\Theta(1)u^*$ where $\Theta(1)$ is the sum of the estimated coefficients of unemployment. This yields:

$$A(L)\Delta p_t = \beta + \Theta(L)u_{t-1} + \gamma z_t + \varepsilon_t \qquad (3.4)$$

where $\beta = -\Theta(1)u^*$. In the absence of supply shocks, the NAIRU is derived as $\hat{u}^* = -\hat{\beta} / \hat{\Theta}(1)$.

Time varying NAIRU: Elmeskov method

A method that, although based on a Phillips curve type relationship, does not rely on the a priori assumption of a stable systematic link between inflation and labour market imbalances is the one developed by Elmeskov (1993). In its original version the method is conceived as exploiting information on wage rather that price developments (the resulting measure of unemployment consistent with constant wage growth is in fact labelled by its proponent as NAWRU), but it can in principle be applied to any indicator of inflation. The NAWRU is currently and routinely applied by the OECD to derive measures of country output gaps, although additional evidence - including judgmental expertise - is used in the process.[16] The NAIRU is computed on the basis of a simplified "accelerationist" version of equation (3.2):

$$\Delta^2 p_t = -\alpha_t (u_t - u_t^*), \qquad (3.5)$$

where Δ^2 is the second difference operator, p is the logarithm of the chosen price (or wage) indicator and α is a positive variable. The two unobservable variables, α and u^*, are identified by constraining them both to be constant across two subsequent observations. In other words, Δu_t^* is assumed to be equal to zero and hence the NAIRU to change only gradually over time. An estimate of α is obtained for any two consecutive periods as:

16 Alternative approaches are currently being investigated by OECD staff, also aimed at assessing the robustness of the results provided by Elmeskov's method. In particular, two methods of estimation of the NAIRU are being considered. The first is based on a Kalman filter applied to a Phillips curve specified in terms of price inflation; the second on a multivariate filter along the lines described in Laxton and Tetlow (1992).

$\hat{\alpha}_t = -\frac{\Delta^3 p_t}{\Delta u_t}$ which is then substituted into (3.5) to give $\hat{u}_t^* = u_t - \left(\frac{\Delta u_t}{\Delta^3 p_t}\right)\Delta^2 p_t$

The obtained measure, smoothed in order to remove erratic movements[17], is a short-run equilibrium indicator, since it represents the unemployment rate associated, in a given period and depending on the recent past evolution of unemployment, with constant wage (or price) growth.

This method, despite its simplicity of construction and the advantage of not requiring a wide variety of data, involves some weaknesses. For example, some potentially important explanatory variables might be omitted. Moreover, there is no estimation involved and therefore no test statistics or statistical measure of the uncertainty associated with the indicator itself. Finally, as α_t is computed as a fraction where the denominator might be close to zero, it can be highly volatile, thus leading to a considerable volatility in the NAIRU itself. This is one of the reasons why the raw measure needs to be filtered.

Time varying NAIRU: the break model

In the "break" model, proposed by Gordon (1982), the NAIRU is allowed to vary, taking different discrete values over time, depending on a number of break dates $\{t_i\}$:

$$u_t^* = \bar{\lambda}' F_t ,$$

where $F_t = (F_{1t},...,F_{nt})$ is a set of dummy variables such that $F_{it} = 1$ if $t_{i-1} < t \leq t_i$ and $F_{it} = 0$ otherwise. The Phillips curve equation (3.2) is reformulated as:

$$A(L)\Delta p_t = \Theta(L)u_t + \lambda' F_t + \gamma z_t + \varepsilon_t \text{, where } \lambda = -\Theta(1)\bar{\lambda} \qquad (3.6)$$

The break points may either be fixed ex ante or estimated. In the latter case, when there is more than one break a sequential algorithm is adopted which estimates one break at a time, treats it as fixed and estimates the next one. The NAIRU is then computed as:

$$\hat{u}_t^* = -\frac{\hat{\lambda} F_t}{\hat{\Theta}(1)}$$

17 An Hodrick-Prescott filter with a smoothing parameter of 25 is applied to the series, in order to reduce the influence of transitory movements in the Phillips curve.

The structural time series/unobserved component method

In this approach, the definition of NAIRU as the level of unemployment at which inflation is constant is explicitly incorporated in the decomposition of unemployment. The process assumed for unemployment is a structural time series model, which decomposes observed unemployment into a trend (the NAIRU), a cyclical (*uc*) and an error (*ue*) component:

$$u = u^* + uc + ue \tag{3.7}$$

Three different issues can be identified for the empirical implementation of this method, and for the related results: i) the choice of the statistical model for u^*; ii) the assumptions concerning the deviation of the unemployment rate from the NAIRU; iii) the explicit introduction of hysteresis effects in unemployment dynamics.

- Choice of the statistical model for u*

The most commonly adopted model for representing the NAIRU is a random walk process:

$$u_t^* = u_{t-1}^* + \tau_t + \eta_t \tag{3.8}$$

where τ_t is a drift and the error term η_t is iid with mean zero and standard deviation σ_η. Most empirical studies assume no systematic trend in the NAIRU, and hence impose τ_t=0.[18]

- Assumptions on the deviation of unemployment from the NAIRU

Empirical analyses frequently assume that the deviation of the unemployment rate from the NAIRU is a white noise process.[19] This assumption amounts to neglecting the cyclical component *uc* in the structural representation (3.7). The system (3.2) and (3.8) can be expressed in state-space form and estimated using the Kalman filter. Estimates of the relevant parameters, i.e. the coefficients of the polynomials in the lag operator A(*L*), Θ(*L*) and γ, and the two variances σ_ε^2 and σ_η^2, can be obtained by maximum likelihood, under the assumption that the two error terms are uncorrelated.

Alternatively, the deviation of the observed unemployment rate from the NAIRU is assumed to follow an autoregressive process (Apel and Jansson, 1997; Rasi and Viikari, 1998; Laubach, 1997). The system is hence augmented by a third equation, which

[18] Staiger at al (1996); Gordon (1996). Recent works, however, produce interesting results allowing the trend in (3.8) to be either constant (linear) or time varying (quadratic). See Laubach (1997).

[19] Staiger et al (1996); Gordon (1996).

formalises the assumption that the observed unemployment rate has a tendency to return over time to equilibrium:

$$u_t - u_t^* = G(L)(u_{t-1} - u_{t-1}^*) + \omega_t \tag{3.9}$$

where $G(1)<1$ and the error term ω_t is *iid* with mean zero and standard deviation σ_ω. The system including equation (3.2) and (3.7)-(3.9) can be solved by applying the Kalman filter to obtain estimates of the unknown parameters and of the time series of the unobserved variable. In the state-space form, the measurement equations are derived from (3.2) and (3.7); the transition equation is derived from (3.8) and (3.9). All shocks are assumed to be mutually uncorrelated and to have constant variances.[20] In principle all the parameters of this system can be estimated by maximum likelihood. However, the empirical implementation of the Kalman filter smoothing algorithm tends to produce a solution in which all the variance parameters are zero apart from the one in equation (3.8) (see, for example, Gordon, 1996). An additional constraint has therefore to be placed on the variation of the NAIRU to avoid that high frequency fluctuations, i.e. jumps from one period to the other in u^*, absorb all the residual variation in equation (3.2). Although the value chosen for the variance of the error term matters only insofar as it determines the high frequency fluctuations of the NAIRU estimates, it influences the standard errors around such estimates. The system might also be extended in order to take into account the relationship between the NAIRU and potential output (Rasi and Viikari, 1998; Apel and Jansson, 1997, 1998). Such a relationship is specified as an Okun equation by which a gap between actual and equilibrium unemployment is reflected in a gap between actual and potential output. In the two papers mentioned this Okun law is imprecisely measured, this being needed to account for shifts in the output gap not originating in changes in the unemployment gap. One of the advantages of the introduction of this additional equation is that the a priori exogenous restriction on the variance of the NAIRU, which is needed for the empirical implementation of both the specifications presented above, is no longer necessary. The other constraints imposed on the whole system by the Okun's law seem to handle, in fact, the variance of the residual.

[20] The assumption of diagonal variance-covariance matrix of the system can also be relaxed, without making the model under-identified, to allow for some non-zero correlation between the error terms. As an example, correlation between shocks to the NAIRU and shocks to cyclical unemployment may occur in the presence of hysteresis effects in the labour market. Apel and Jansson (1998), however, find that the assumption of orthogonality between the residuals cannot be rejected.

- Introducing hysteresis effects

Model (3.2), (3.7)-(3.9) allows also to explicitly take into account the possibility that hysteresis effects in unemployment affect the pattern of the estimated NAIRU (Laubach, 1997). Equation (3.9) may be rewritten as:

$$u_t^* = u_{t-1}^* + \psi(u_{t-1} - u_{t-1}^*) + \eta_t \qquad (3.10)$$

where the introduction of the persistence term replaces the deterministic drift τ_t and the coefficient ψ is assumed to be bounded between 0 and 1. In the extreme case in which ψ=0, (3.10) corresponds to a random walk. Conversely, when ψ=1 the term u^*_{t-1} cancels out and the NAIRU is only defined as lagged current unemployment plus a shock. Constant inflation, in this case, would be compatible with any level of unemployment.

4 The euro area NAIRU: some preliminary estimates

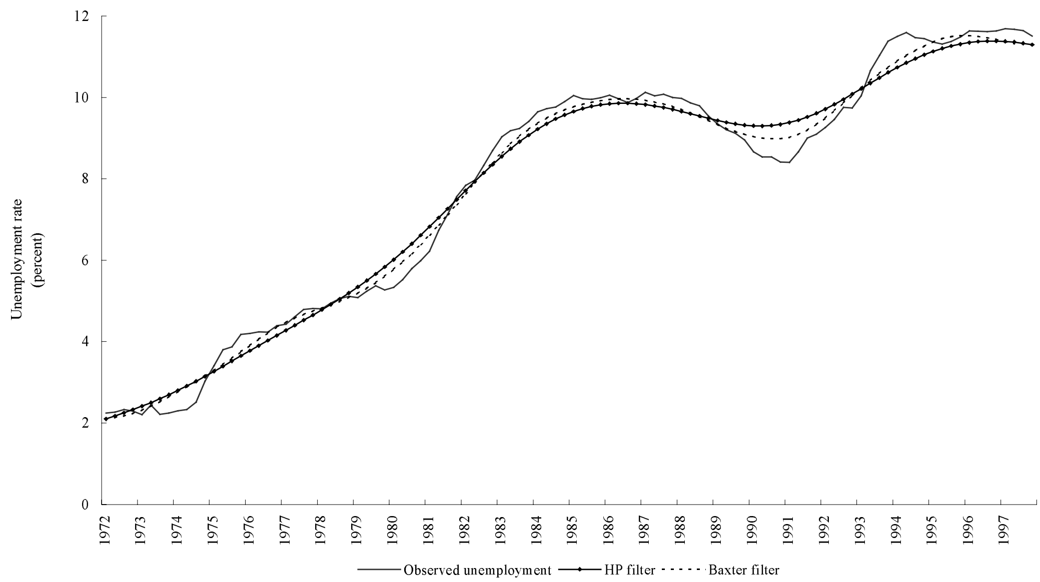
4.1 Filtering techniques

Univariate filtering techniques, although quite easy to implement, do not use any information other than the statistical properties of unemployment and make no reference to the inflationary process, which has a primary role in the conceptual definition of NAIRU. Despite this limitation, the smoothing process involved in such methods is consistent with the idea that the determinants of the NAIRU change only slowly over time. This is probably one of the main reasons why filtered unemployment series are often found to be significantly correlated with inflation dynamics (see, for example, Bank of England, 1999).

On the ground that the empirical evidence supplied by this approach might shed some light on the non-cyclical component of unemployment in the euro area, a standard HP filter (with λ = 1600) and a bandpass filter based on the methodology proposed by Baxter and King (1995) have been applied.[21]

[21] As for the HP filter, in order to overcome the problem of end-of-sample imprecision of the estimates, two additional years of data have been used to filter the unemployment time series.

Figure 4.1 NAIRU estimates based on filtering techniques



Despite being founded on different principles, the two techniques produce strikingly similar results, as figure 4.1. clearly shows.[22] Similar results, which are not shown here, are also supplied by the implementation of the simple bivariate version of the HP filter described in section 3, in which the smoothing parameter λ is chosen to maximise the fit of the Phillips curve.[23]

4.2 Methods based on the Phillips curve

The analysis presented here addresses the issue of estimating the NAIRU using the available information on price, wage, productivity and unemployment developments in the euro area. The basic framework is the simple Phillips curve already described in the previous section, which captures a disequilibrium adjustment mechanism: inflation depends on its past values, on the tightness of the labour market and other factors potentially affecting its response to demand pressures:

$$A(L)\Delta p_t = \Theta(L)(u_t - u_t^*) + \gamma z_t + \varepsilon_t \qquad (4.1)$$

22 Baxter and King method is a bandpass filter, which passes frequencies corresponding to between 8 and 32 periods, which is a typical business-cycle frequency range when one uses quarterly data. The Hodrick-Prescott filter is derived by minimising the sum of the squared deviations of the unemployment rate from its trend, subject to a smoothness constraint that penalises squared variations in the growth of the trend.

23 The additional finding which emerges from this experiment is that the best fit is provided by a filtered series that approximates a linear trend, corresponding to a smoothing parameter that tends to infinity. The improvement in the statistical fit of the Phillips curve due to the increase in λ, however, is almost negligible, and the results of such an experiment are not presented here.

Since expression (4.1) can be thought of as reduced form of a structural wage-price setting model, it is in principle possible to express it either in terms of wages or of prices. For wage inflation, we follow Gordon (1998) and focus on the rate of growth of trend unit labour costs, i.e. wages divided by trend productivity, on the ground that this is, after all, the variable relevant to pricing decisions.[24] For price inflation, we model both the change in consumption deflator and the one in GDP deflator, which excludes the direct impact of import prices. For a meaningful measure of the NAIRU to exist, a long-run homogeneity restriction is imposed on the lagged coefficients of the inflation indicator. In (4.1), such coefficients capture both the influence of past inflation behaviour and of expectations on current price setting. The use of an explicit measure of expected inflation, which would allow a formal distinction between inflationary expectations formation and the inertia of the inflationary process, is not possible due to the unavailability of data. The tightness of the labour market is proxied by current and past values of the unemployment gap.[25] The presence of several lags of such a variable automatically allows inflation to depend on both its level and change. The relationship linking price dynamics to the unemployment gap is assumed to be of a linear form. This formulation is consistent with a battery of preliminary tests for the presence of non-linear effects, which has provided evidence that didn't allow rejecting the hypothesis of a linear Phillips curve.[26]

Constant NAIRU

If the NAIRU is assumed to be constant over the entire sample period, equation (4.1) is reformulated as:

$$A(L)\Delta p_t = \alpha + \Theta(L)u_t + \gamma z_t + \varepsilon_t \qquad (4.2)$$

[24] We consider trend productivity growth, i.e. a smoothed version of actual productivity growth, on the basis of the idea that productivity gains are not translated into wages in the very short run.

[25] One could argue that the inflationary effects of tight demand plausibly occur with a lag. The issue of whether or not to include among the regressors of the Phillips curve the current unemployment gap has in fact been debated in the empirical literature. Here, we chose to adhere as much as possible to the standard textbook formulation of the relationship, which incorporates the current value. The empirical results, however, do not change with the exclusion of such a variable.

[26] Non-nested tests of the linear Phillips curve against an alternative version in which unemployment entered in logarithms rather that in levels were performed, such as the Cox test, its adjusted version derived by Godfrey and Pesaran (1983), the J and the encompassing tests. Akaike and Schwarz choice criteria based on the maximised log-likelihood function of the two alternative models were also evaluated. We are aware of the limitation of considering only the logarithmic functional form as an alternative specification to a linear one. The class of nonlinear Phillips curves is vast and could include, for example, the possibility that the speed of adjustment depends on the level of the equilibrium rate of unemployment, or that supply shocks have asymmetric effects.

and the NAIRU is computed as the OLS estimate of the ratio of α to the sum of the coefficients of the unemployment term.

Table 4.1 reports the results obtained using quarterly data on total unemployment rate, trend unit labour costs, consumption deflator, GDP deflator for the euro area, referred to the period 1972:1 1997:4. The regressions also include, according to the specific indicator chosen as dependent variable, other explanatory factors: the import deflator when inflation is measured by the change in consumption deflator and unit labour costs when it is the change in GDP deflator. Each regression includes four lags of the inflation indicator, contemporaneous and lagged (four lags) values of unemployment and four lagged values of the additional explanatory factors. The second column of the table reports the estimated sum of coefficients of current and lagged unemployment, which capture the response of price or unit labour cost changes to the cyclical situation in the labour market and can therefore be thought of as a measure of aggregate real rigidity. The third column reports the NAIRU figures implicit in the estimated Phillips curve. The standard errors are derived using the "delta method", given that the NAIRU is computed as a non-linear function of the regression coefficients.[27] The last column of the table contains the 95 percent confidence interval obtained on the basis of such standard errors.

Table 4.1

dependent variable	$\bar{R}^2$	Θ(1) (s.e)	NAIRU (s.e.)	95% confidence interval
consumption deflator	0.94	-0.02 (0.01)	9.1% (1.57)	5.9% - 12.2%
GDP deflator	0.89	-0.02 (0.01)	8.6% (1.59)	5.5% - 11.8%
ulc* growth	0.77	-0.09 (0.03)	8.4% (0.67)	7.1% - 9.8%

Overall, the fit of the equations, whatever the indicator modelled, is quite high. The implied unemployment rate consistent with stable inflation over the period is in the range of 8.4 – 9.1 per cent. The estimates show a strong similarity across the three

27 The delta method is a technique for constructing asymptotic standard errors for functions of estimated parameters. Suppose that $F = F(\theta)$ is a function of the parameters θ of the model and assume it is first order differentiable. The estimate of F and its corresponding variance matrix can is computed according to the formulae:

$$\hat{F} = F(\hat{\theta})$$

$$\hat{V}(\hat{F}) = \hat{\sigma}^2 \left[\frac{\partial F(\theta)}{\partial \theta}\right]_{\theta=\hat{\theta}} \hat{V}(\hat{\theta}) \left[\frac{\partial F(\theta)}{\partial \theta}\right]'_{\theta=\hat{\theta}}$$

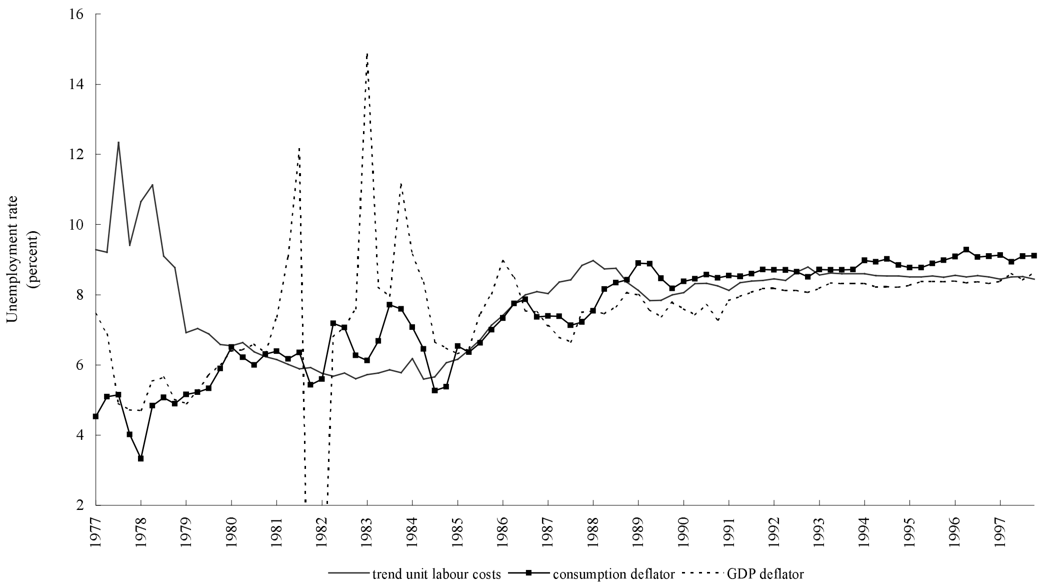
where $\hat{\theta}$ is the maximum likelihood estimate of the parameters and $\hat{V}(\hat{\theta})$ a consistent estimator of its variance matrix. See also Rao (1973). A potential drawback of this method lies in the fact that it approximates the distribution of the estimated NAIRU, which is a ratio of coefficients, by a normal distribution.

specifications, supporting the idea that wage behaviour does not play much of an independent role in the inflation process (Gordon, 1998). However, their standard errors are sensitive to whether the Phillips curve is modelled in terms of price or unit labour cost inflation: the 95% confidence band for the former is almost double the one computed for the latter (six versus three percentage points). This result stems from the fact that the direct effects of labour market tightness account for a relatively small amount of the movements in quarterly price growth.

In order to verify the reliability of the hypothesis of a constant NAIRU over the period, the estimated relationships have been subject to a thorough stability analysis. Several pieces of evidence potentially available to detect structural shifts in the Phillips curve have been considered. As the timing of the potential breaks cannot be established on a priori grounds, a test aimed at detecting instability of a general form has been performed as a first attempt (Hansen, 1994). The test is approximately a Lagrange multiplier test of the null hypothesis of constant parameters against the alternative that they follow a martingale and it has the advantage of detecting the coefficients responsible for the eventual break. The results provide only limited and not clear-cut evidence in favour of structural shifts in the equation. Although overall stability of the entire equation seems to be a common feature of the three different specifications considered here, some of the coefficients show some sign of instability over the sample period.

A further investigation of the properties of the estimated relationships has been conducted by means of simple CUSUM tests, essentially aimed at detecting time variation in the intercept, and CUSUMSQ tests, focused on the stability of the variance. Both types of exercise provide some, although not striking, sign of unstable behaviour of the equations in the 80's. An interesting piece of evidence has been derived from the recursive estimates of the NAIRU level implicit in the parameters of the Phillips curve, shown in Figure 4.2. If one neglects the initial years, which reflect the limited number of observations on which the recursive estimates are constructed, three elements emerge quite clearly. First, comparing the beginning, say 1980, and end estimates reveals an upward shift in the NAIRU (from about 6 to 8 per cent). Second, such a shift takes place in the first half of the eighties, whatever the indicator chosen as a measure of inflation. Finally, the implied NAIRU seems to have remained quite stable over the last decade.

Figure 4.2. NAIRU recursive estimates based on Phillips curves



Overall, the evidence provided above, however mixed, and the upward movement in the observed unemployment rate do not make the idea that the NAIRU in the euro area can be treated as a constant a very realistic and appealing one. Therefore, one should allow such a measure to shift over time, and the issue is finding the best way to do that.

Elmeskov method

A very simple method, often adopted in the literature (see, for example, Ball, 1996) for allowing the NAIRU to vary over time exploiting information on prices and unemployment is the one developed by Elmeskov (1993). Although the author estimates the unemployment rate consistent with stable wage inflation, the technique can be applied to any accelerationist Phillips curve, whether expressed in terms of unit labour costs, wages, or price inflation.

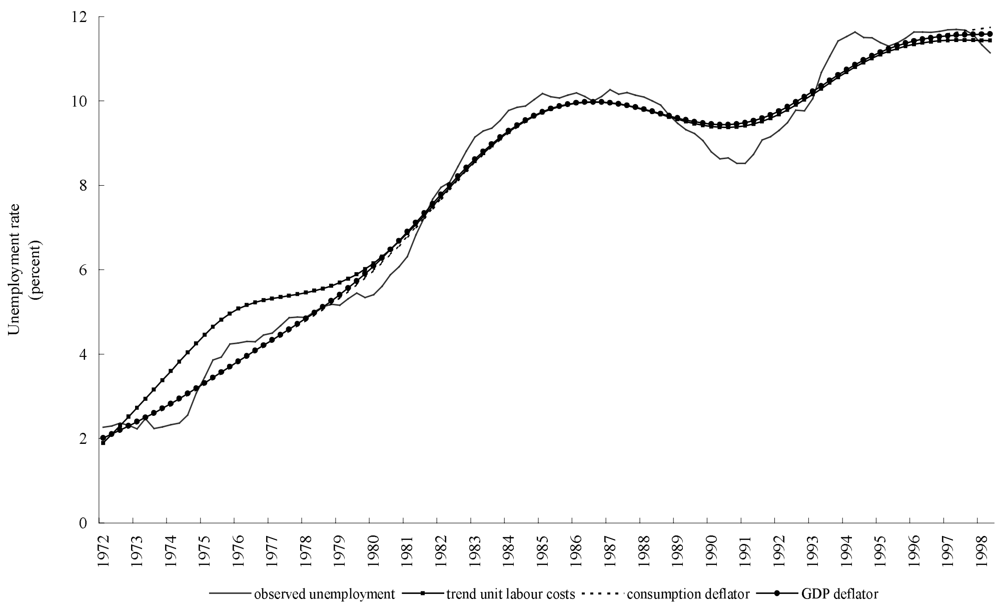
In order to obtain a measure of the NAIRU comparable to the one commonly computed for European countries by the OECD[28], this technique, described in section 2, has been applied to a simplified version of equation (4.1), which does not include lagged terms of

[28] The OECD utilises NAIRU estimates derived with this methodology for a variety of purposes, from the calculation of output gaps, to the construction of government structural balances, to measure structural unemployment. The figures obtained applying Elmeskov method to the aggregate euro area data seem to be consistent with the ones reported by the OECD in several studies (see Giorno, Deserres, and Sturm, 1997). For example, OECD estimates of the NAIRU for 1997 are equal to 10.6 for Italy, 10.2 for France, 9.6 for Germany, 19.9 for Spain, 11.6 for Belgium. The corresponding value for the whole euro area resulting from our analysis is of 11.4.

inflation and unemployment nor additional factors. The NAIRU estimate for a given period has been derived from unemployment and the change in inflation (wage or price) in that period and in the previous one. The obtained series have then been mildly smoothed with an HP filter with parameter equal to 25, in order to reduce the influence of supply shocks and other transitory shifts in the Phillips curve. The resulting time varying NAIRU for each of the three indicators chosen to measure inflation are presented in figure 4.3, together with the series of observed unemployment.

The three NAIRU indicators show a tendency to follow actual unemployment rather closely, thus suggesting that the rise in European unemployment is mainly explained by a rise in "structural" unemployment. However, as the NAIRU measures are not derived by standard estimation methods, they elude the possibility of testing and can hardly be subject to empirical evaluation.

Figure 4.3. NAIRU estimates based on Elmeskov method



"Break" NAIRU

In the Phillips curve specification summarised by equation (4.2), the NAIRU mainly collapses in the intercept. Statistical procedures can hence be implemented to determine the location of breaks in such a coefficient and, on the basis of the results obtained, dummy variables can be included among the regressors to capture the located shifts. The

NAIRU can therefore be modelled as having discrete jumps at certain points in time and as being constant between such points.
Clearly, this solution is not immune from criticism. It does not take into account the potential source of uncertainty deriving from the possible stochastic nature of the time varying NAIRU. The breaks are in fact treated as occurring non randomly and, once occurred, as if they were known with certainty. An extension of this model would be one in which the NAIRU switches stochastically between different regimes like the Markov switching regression model proposed by Bianchi and Zoega (1998). The confidence intervals associated with the estimated NAIRU would be different from those for the deterministic break model presented below, since they would incorporate the additional uncertainty of not knowing the current regime.[29]

In practice, it is not easy to determine the exact timing of the breaks. The existing empirical literature mainly relies on the a priori assumption of equidistant breaks (see, for example, Staiger et al, 1996) or, more rarely, on sequential estimation algorithms. The method adopted here is based on the implementation of Goldfeld and Quandt (1973) switching regression testing procedure.[30] For each of the three specifications of the Phillips curve the results provided by this test support the existence of at least one break. The estimated timing of switches in regimes (beginning of 1985, beginning of 1981 and end of 1984 for the three alternative specifications, respectively) seems to be consistent with the empirical evidence deriving from the recursive estimates of the NAIRU presented above. Chow tests for structural breaks occurring at such dates confirm the presence of a change in regime. The hypothesis that further changes occurred over the sample period is rejected by the data.

[29] This source of uncertainty is instead taken into account in the time varying model presented in the next section, where the NAIRU is explicitly treated as an unobservable stochastic parameter.

[30] Assume that the observations on a dependent variable y, related to a set of independent variables $x_1,\ldots,x_p$, are generated by two distinct regimes:

$$y_i = x_i^{'}\beta_1 + u_{1i} \quad \text{for } i \le i^*$$

$$y_i = x_i^{'}\beta_2 + u_{2i} \quad \text{for } i > i^*$$

where it is generally assumed $\beta_1 \neq \beta_2$. The two regimes are estimated by maximising the likelihood function conditional on i* and then choosing as the most likely estimate of i* the value that maximises them. The null hypothesis that no switch took place at time i* is finally tested on the basis of a likelihood ratio test given by:

$$\text{LR} = \frac{\hat{\sigma}_1^{i^*} \hat{\sigma}_2^{(n-i)^*}}{\hat{\sigma}^n}$$

where $\hat{\sigma}$ is the estimated standard deviation of the residuals from a regression over the whole sample period.

"Break" NAIRU measures have then been computed for the euro area, allowing the intercept in the Phillips curve to switch from one level to another after the break has occurred.

Table 4.2 and figures 4.4 to 4.6, which show the results, suggest a few general considerations.

- Since the beginning-mid eighties the NAIRU in the euro area seems to have risen on average by almost 4 percentage points.

- The precision of the "break" estimates[31] is higher, although still not satisfactory for the Phillips curves measured in terms of prices, with respect to the specification with constant NAIRU, as the 95% confidence bands show.

- Finally, the most precise estimate is obtained, as in the case of constant NAIRU, for the trend unit labour cost Phillips curve. This is particularly true for the second regime, in which the 95% confidence band covers values included between 8.8 and 10 percent, a remarkably low range compared both to the "constant" hypothesis and to the main findings in the empirical literature.[32]

Table 4.2

dependent variable	$\overline{R}^2$	Θ(1) (s.e.)	NAIRU (s.e.)	95% confidence interval
consumption deflator	0.94	-0.04 (0.01)	5.9% (1.15)	3.6% - 8.2%
			10.2% (0.87)	8.5% - 11.9%
GDP deflator	0.89	-0.06 (0.01)	5.5% (1.06)	3.3% - 7.6%
			9.7% (0.68)	8.3% - 11.0%
ulc* growth	0.79	-0.22. (0.05)	5.9% (0.49)	5.1% - 6.9%
			9.4% (0.33)	8.8% - 10.0%

31 The standard errors have been derived, also in this case, applying the delta method.
32 Staiger et al (1996) find the typical 95% confidence interval for the NAIRU in the US being 5.1 to 7.7%.

Figure 4.4. One-break estimate (1985:1) of NAIRU
(price-price Phillips curve with consumption deflator)
NAIRU 1st regime = 5.86% (t stat. = 5.1)
NAIRU 2nd regime = 10.2% (t stat. = 11.7)

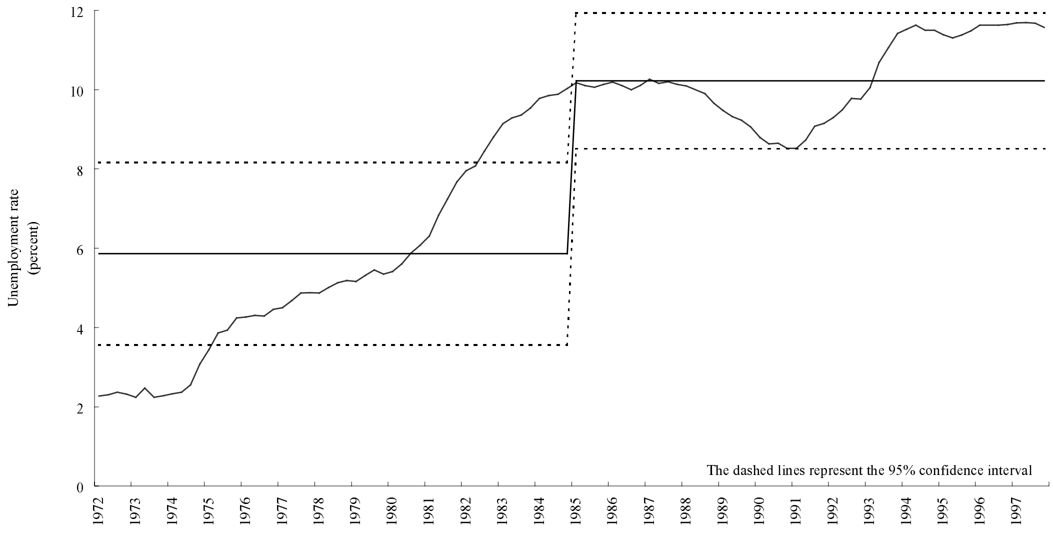


Figure 4.5. One-break estimate (1981:1) of NAIRU
(price-price Phillips curve with GDP deflator)
NAIRU 1st regime = 5.5% (t stat. = 5.12)
NAIRU 2nd regime = 9.7% (t stat. = 14.2)

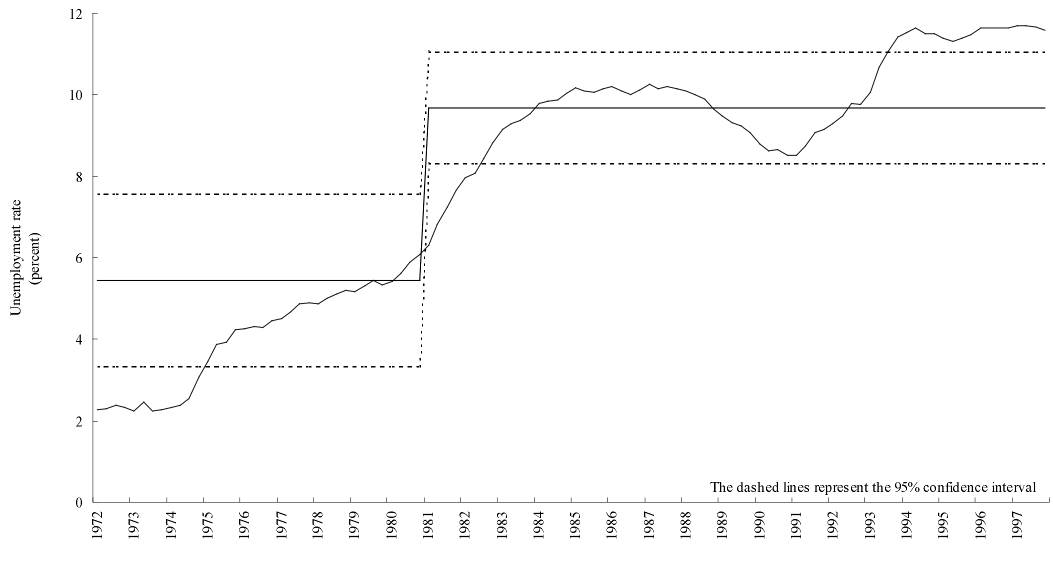
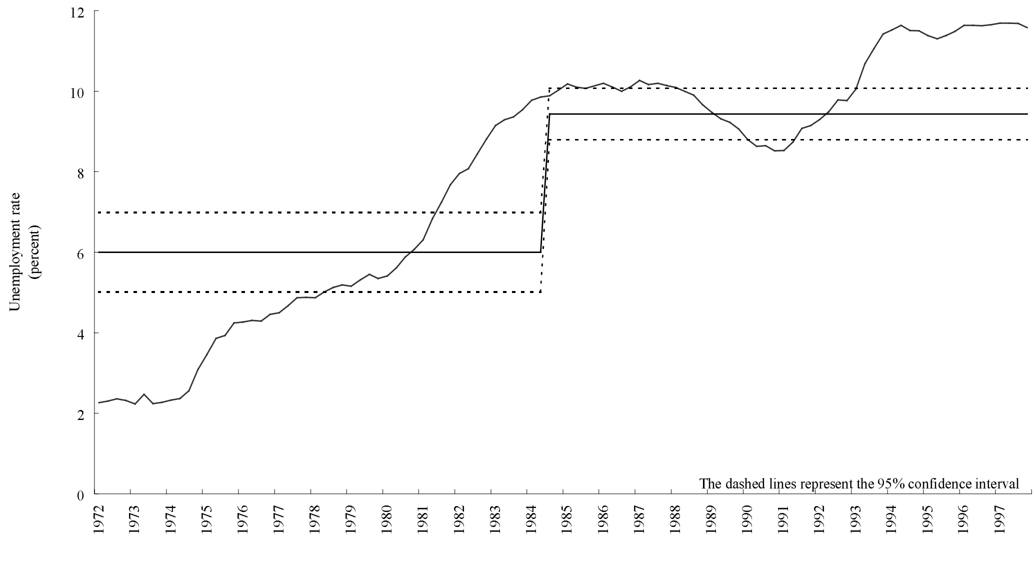


Figure 4.6. One-break estimate (1984:3) of NAIRU
(trend unit labour cost Phillips curve, inflation measured by consumption deflator)
NAIRU 1st regime = 5.9% (t stat. = 12.01)
NAIRU 2nd regime = 9.4% (t stat. = 28.94)



5 The time-varying parameters approach

The time-varying NAIRU is just an extension of the methods proposed until now but still within the same general approach. The basic inflation equation is supplemented by a specific assumption for the law of movement of the NAIRU. This law of motion must ensure a resulting estimate with behaviour not far from that of observed unemployment, but smoother variations. This goal is reached assuming that the NAIRU moves over time as a random walk.

5.1 The empirical model

In the light of the previous discussion, the model underlying the time-varying NAIRU approach comprises a reduced form Phillips curve like (5.1a) or a wage-price Phillips curve such as (5.1b). The first one links inflation Δp (alternative definitions of it being used in the analysis) with the unemployment gap u-u^* and other relevant variables z. The second equation distinguishes between movements in wages and prices, forcing the appearance of a productivity term, q. In the two equations, the variable z may include factors far from irrelevant. In general terms, it comprises all the supply-side factors affecting inflation not already captured by changes in unemployment, probably including business-cycle changes in marginal costs. But other factors may be included in this variable, such as speed-limit effects (i.e., first differences of unemployment) or, in the presence of a credible monetary policy, stationary inflation targets. Variations on this basic framework include using diverse measures of inflation, differing degrees of dynamic homogeneity in inflation and alternative specifications for z.[33]

$$A(L)\Delta p_t = -\Theta(L)\left(u_t - u_t^*\right) + \gamma' z_t + \varepsilon_t \qquad (5.1a)$$
$$A(L)\Delta w_t = B(L)\Delta p_t + C(L)\Delta q_t - \Theta(L)\left(u_t - u_t^*\right) + \gamma' z_t + \varepsilon_t \qquad (5.1b)$$

It is obviously necessary to disentangle somehow the variable u^* in this equation. Assuming a specific law of motion for it does this. In the case at hand, a very simple but very general random walk is assumed, as in equation (5.2). As already explained, this is a convenient expression due to its intrinsic simplicity but with the potential for coping with more complex dynamic processes.

$$u_t^* = u_{t-1}^* + \eta_t \qquad (5.2)$$

Equations (5.1)—either (5.1a) or (5.1b)—and (5.2) define a recursive Kalman filter, which can be solved to derive a maximum-likelihood measure of the NAIRU. Equation (5.1) is the measurement equation; equation (5.2) is the transition equation. The model is equivalent to one in which the parameter multiplying the constant was time varying. This means that the variability of the time-varying parameter must be constrained, in order to avoid a vanishing residual term in (5.1). It is of course an issue what the variability of the NAIRU may be on *a priori* grounds. The standard assumption is to give a rather low variability to the process, in view of the links assumed to exist between this measure and the inner structure of labour markets. On the other hand, there is a growing stream of literature pointing to recurrent shocks hitting the labour market at relatively high frequencies. In view of this, a pragmatic approach was chosen, and a variance of the NAIRU not far from that of observed unemployment was selected, the two expressed in first difference. It is well known that the size of the imposed variance matters for the final shape of the estimated NAIRU (see e.g. Gordon, 1997), this aspect being considered as one of the main weaknesses of the method. In order to assess this problem, a number of different variances were tested as a robustness test. It was found that the range of admissible variances – i.e, those providing estimated NAIRUs with a sufficient degree of smoothness and plausibility – was larger than expected.[34]

33 The estimations were not systematically run under the assumption of dynamic homogeneity, although great care was taken to demean those variables for which this was necessary.

34 This was a consequence of the chosen estimation strategy, to be detailed below, which allowed for a different *ex-post* variance. Actually, it was found that sample variances (i.e., the final variance of the first difference of the estimated NAIRU) were about one-third the *a priori* one. This gave a correspondingly larger range of variation in the initial assumption.

5.2 The estimation strategy

The model (5.1)-(5.2) can be estimated by iterating on the Kalman filter solution until the parameters have reached a value that maximises the likelihood. This approach was initially used in the context of this exercise, but a simpler approach was finally taken. The method is a derivation of Stock's proposal (Stock, 1999) to derive the NAIRU following a two-step approach. In the first step, an ordinary least squares regression is run on a standard Phillips curve specification, with the inclusion of a constant (which gives the average NAIRU over the sample). In a second step, the derived constant in the regression—plus the residual—is taken as input for a Kalman filter in which the NAIRU is assumed to be a random walk, whose variance has to be chosen. The procedure is simple to implement and very fast, its main drawback being that it is not a full optimisation of the filter.

This method is useful if the value of the NAIRU is never far from the mean over the whole sample (which is probably the case in the US). However, given the strong upward trend in euro area unemployment, this assumption is untenable. The approach was accordingly amended to include iterations on the two steps, until maximum likelihood is reached. It turned out that the number of iterations was relatively low as long as the starting point was good enough. This entailed an initial least squares regression with a significant unemployment gap in it, as otherwise the initial estimate of the NAIRU was too imprecise. A grid search of possible initial off-model estimates of the NAIRU, necessary for the first iteration, was tried: alternatives ranged from a constant NAIRU to others closely tracking observed unemployment. It was found that the Hodrick-Prescott filter gave a good starting point, other filters presenting minor problems.[35] Each time the procedure converged, it was ascertained—in an appropriate neighbourhood of the solution—that it was not a local minimum.[36]

One noteworthy feature of this method is that adding dynamic terms in the underlying equation has sometimes an adverse impact on the final estimate of the NAIRU. It was ascertained that adding lags to the unemployment gap itself, while adding nothing in terms of goodness of fit, worsened the precision with which the sum of its parameters was estimated. In the same vein, adding lags of auxiliary variables, when there was some degree of auto-correlation among them, prevented convergence to the solution. In view of

35 As, for instance the loss of initial and last observations in the Baxter-King filter.

36 At each instance, there was no further iteration giving a better likelihood.

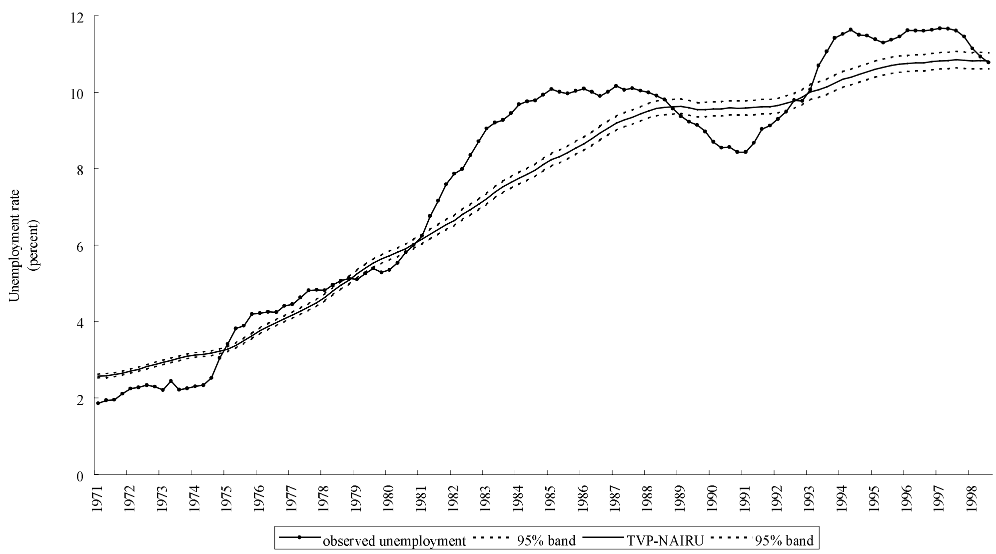
these problems, all regressions have been kept as simple as possible, both in terms of lags or number of regressors included in the measurement equation.

5.3 The resulting estimates

Reduced form Phillips curves, GDP deflator versus consumption deflator

Chart 5.1 presents one possible estimate of the NAIRU derived from a Phillips curve.[37] It shows the NAIRU resulting from a reduced form Phillips curve with four lags of each variable. Inflation is measured by (log) changes in the consumption deflator. Alternative measures were calculated using the GDP deflator inflation. Both measures were found to be similar, with only occasional differences. The underlying equations were, though, slightly different: the consumption equation has a clearer role for the unemployment gap, somewhat muted in the case of the GDP deflator equation. Only one lag of the unemployment gap enters the measurement equation, although results were not much different with up to four lags. The contemporaneous unemployment gap was barely significant, and its inclusion did not alter results. Imposing dynamic homogeneity altered somewhat the resulting estimate, but in no fundamental way.

Figure 5.1. TVP estimate of NAIRU
(reduced form Phillips curve, inflation measured by consumption deflator)



The estimates tell much the same story. Unemployment was marginally below the NAIRU at the start of the seventies, after which both raised broadly in line. It is not until the

[37] Bands around the NAIRU do not synthesise asymptotic uncertainty, because of the random-walk hypothesis: they correspond to 1-step-ahead forecast error variance. (I.e., they embrace the 95% confidence region of where the next value of the NAIRU may lie.) In this sense, they cannot be compared with (asymptotic) bands shown elsewhere in the paper.

eighties that a clear increase in the unemployment gap occurs, the worst episode taking place from the beginning of the decade until—roughly—the mid-eighties. The unemployment gap seems to disappear at the turn of the decade. Finally, the unemployment gap increases again strongly during the early nineties, to be reduced afterwards. According to these measures, the current level of unemployment may not be far from the NAIRU, although probably still above it.

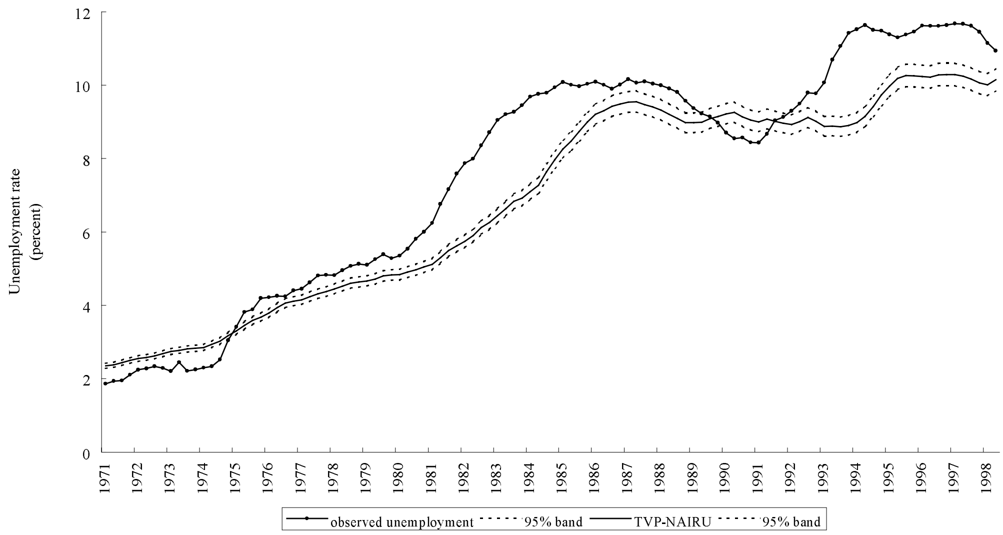
Wage-price Phillips curves

Results obtained from wage-price curves did not differ widely from reduced-form Phillips curves. The benchmark relationship estimated includes terms in inflation measured by the consumption deflator or the GDP deflator, and also productivity terms. Figure 5.2 depicts the NAIRU derived from a real consumption wage equation. This equation embodies dynamic homogeneity with respect to prices, either consumption or GDP deflator, although not having it did not alter results significantly. Results were also pretty similar if instead dynamic homogeneity with respect both to prices and productivity was imposed.

One important aspect of these equations is the addition of a long-run component with real unit labour costs in levels. This last variable was meant to capture possible effects of changes in marginal costs not correctly measured by the NAIRU. Assuming that steady-state real unit labour costs are constant[38], this approach means in fact that we may be catching business cycle effects of real marginal costs. This long-run component has in general only a secondary influence in the wage equations with consumption inflation, but is important when GDP deflator inflation is used. This lends support to the idea that we may be using two different variables for the measurement of the business cycle: the unemployment gap and the real-marginal-cost gap. In that sense, a wage curve in terms of consumption deflator inflation including no marginal cost measure may be claimed to perform a more accurate identification of the NAIRU.[39] As was the case with the Phillips curves, imposing varying degrees of dynamic homogeneity altered somewhat the results, but in no substantive way.

[38] Something far from evident for European countries, though, given the behaviour of the labour share.
[39] But not necessarily providing a better estimate.

Figure 5.2. TVP estimate of NAIRU
(wage-price Phillips curve, inflation measured by consumption deflator)



Real unit labour cost equations

Another potential set of equations that could be interesting to test is real unit labour costs equations, i.e. equations linking inflation and unit labour costs. A notable point in these equations is the rather muted role played by unemployment. In fact, in some of the estimations unemployment was barely significant, and correspondingly the starting point for the recursions of the filter was rather poor. It was clearly the case that these equations could be better understood as marginal-cost equations, with unemployment playing only a secondary role.

6 Evaluating the alternative NAIRU measures

One potential problem with NAIRU estimates derived from the analysis of area-wide data is the potential for serious aggregation biases which could lead to imprecise and inaccurate measurement of the concept. This problem stems from the lack of uniformity across member countries' labour markets, as mentioned in the introduction. The obvious alternative is a country-specific analysis of unemployment. However, in the case at hand, since no explicit structural analysis is attempted, there is some ground to justify an aggregate approach: namely, that it may be sufficient to measure inflation developments. NAIRU analysis in the literature is used for many purposes, but regarding monetary policy there are two worth isolating: NAIRU as an indicator of inflationary pressures and

NAIRU as a component in the derivation of potential output and the output gap. Although both approaches cannot be considered to form the core of the analysis herein, some reference to them is probably not out of place.

6.1 Inflation forecasting performance

Tables 6.1 and 6.2 collect some data on the inflation forecasting performance of the unemployment rate and the unemployment gap based on some of our NAIRU estimates. Results were obtained not through the careful fine-tuning of equations providing optimal forecasts of inflation, but as checks of the performance of sets of simple equations, very often widely different from the equations from which the corresponding NAIRUs were derived. The procedure entailed performing out-of-sample inflation forecasts based on Phillips-curve-like specifications.

Table 6.1 collects data on errors in forecasting inflation for the period 1992Q1 to 1997Q4, with models estimated up until the last period before each exercise was carried out. The models comprise an auto-regressive model for inflation, with four lags and a constant, as the benchmark; a standard Phillips curve, corresponding to the constant NAIRU; a break-NAIRU equation and the TVP-NAIRU. In all cases, unemployment and - whenever relevant - the NAIRU have been modelled as random walks. As is now standard (see Staiger et al, 1997, among others), each forecast has been performed recursively for as many steps ahead as the sample period allowed, a number of *n*-step ahead forecast errors being collected in the process. The table shows mean errors, absolute mean errors and root mean square errors for respectively 1, 4 and 8-step ahead forecasts.

Table 6.2, on the other hand, gathers data from models in which current and lagged information is regressed on inflation *n* periods ahead. In this case, each forecast is a non-recursive one but each model has a different projection horizon (see Staiger et al, 1997). The same error statistics are collected - and averaged - as above. Inflation is always measured using the consumption deflator. It is to be stressed again that these models do not pretend to be optimal forecasting models, their only aim being to test in a robust and comparable manner whether unemployment in general and the NAIRU in particular is a useful tool in the inflation forecasting toolkit.

Table 6.1. Out of sample recursive forecast

	mean error	mean absolute error	root mean squared error
steps ahead		AR(4) inflation	
1	-0.08	0.17	0.23
4	-0.24	0.26	0.32
8	-0.47	0.47	0.49
steps ahead		Phillips curve	
1	0.01	0.16	0.21
4	-0.01	0.13	0.19
8	-0.01	0.12	0.15
steps ahead		Break model	
1	0.02	0.15	0.21
4	0.03	0.12	0.17
8	0.04	0.13	0.15
steps ahead		TVP NAIRU	
1	-0.01	0.15	0.21
4	-0.06	0.16	0.24
8	-0.08	0.25	0.33

Table 6.2. Moving horizon forecasts

	mean error	mean absolute error	root mean squared error
Periods ahead		AR(4) inflation	
1	-0.08	0.17	0.23
4	-0.28	0.29	0.34
8	-0.54	0.54	0.55
Periods ahead		Phillips curve	
1	-0.02	0.17	0.22
4	-0.08	0.16	0.21
8	-0.18	0.24	0.30
Periods ahead		Break model	
1	-0.02	0.17	0.22
4	-0.05	0.16	0.21
8	-0.22	0.27	0.30
Periods ahead		TVP NAIRU	
1	-0.03	0.17	0.22
4	-0.23	0.25	0.33
8	-0.50	0.50	0.61

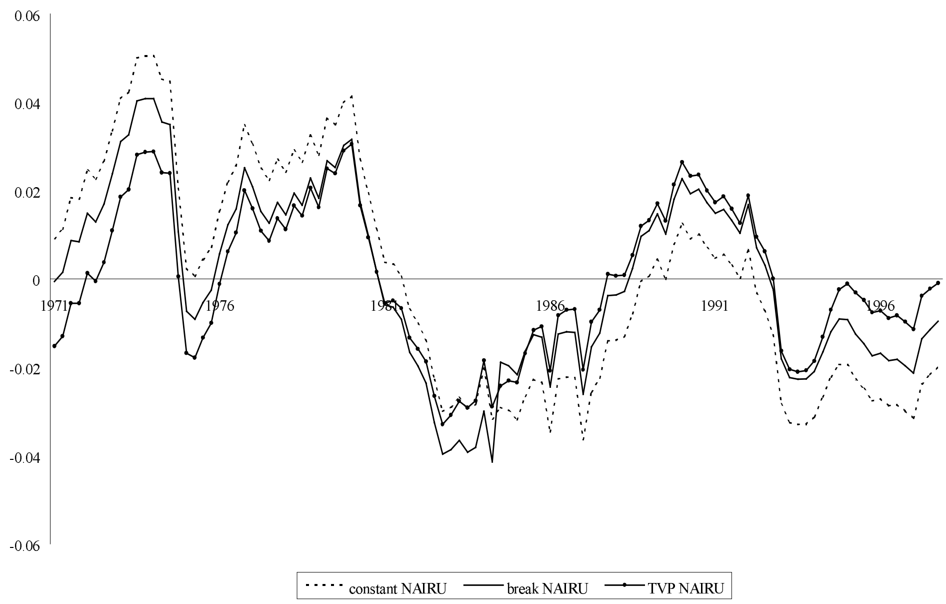
The tables hint at a more than casual link between inflation and the unemployment gap. In all cases, the Phillips curve outperforms a simple AR model of inflation, sometimes by a wide margin. Results not reported here strengthen this conclusion, as the finding is robust to changes in the definition of inflation or in the specification of the underlying system. More complex measures of the NAIRU - such as the break-NAIRU, the TVP-NAIRU or (unreported) filtered NAIRUs - provide mixed results, being in general not worse than the benchmark but somewhat worse than a simple Phillips curve. This last result, though, is not robust enough to warrant firm conclusions. The TVP-NAIRU, for instance, does a better job of tracking inflation when it is measured by the GDP deflator, and shows some worsening just at the end of the sample: we may yet again be facing an imprecise last-period estimate of the smoothed series. Nevertheless, the striking fact arising from this exercise is the apparent but robust link between inflation and the unemployment gap, a stylised fact in the US and other economies which seems to be at work at the area level.

6.2 Derivation of output gaps

Another useful testing ground for our measures of the NAIRU rests on a widely used indicator of future inflation: the output gap. Although output gaps have been calculated in the literature following a number of different methods, one of the most common is the so-called production function approach. Production functions link observed output and observed use of factor inputs, a measure of potential output—and hence of the output gap—being derived by replacing the latter by 'normal uses' of production inputs. An exercise of this kind can be attempted using the NAIRU measures presented beforehand.

A number of shortcuts are necessary to compute potential output, as it is not our goal to perform a sophisticated exercise. Hence, it is assumed that technology is given by a Cobb-Douglas production function with constant returns to scale. The wage-share parameter is taken to be the sample average for the wage income share in GDP, while total factor productivity is a log-detrended version of the Solow residual. It is further assumed that capital stock is always optimally used. Potential output is derived by replacing observed employment by the NAIRU and computing the corresponding output. The resulting measures have been re-scaled to provide an output gap with an in-sample mean of zero. Results for the constant-NAIRU, break-NAIRU and TVP-NAIRU are shown in graph 6.1.

Figure 6.1 Output gap derived from different NAIRU measures



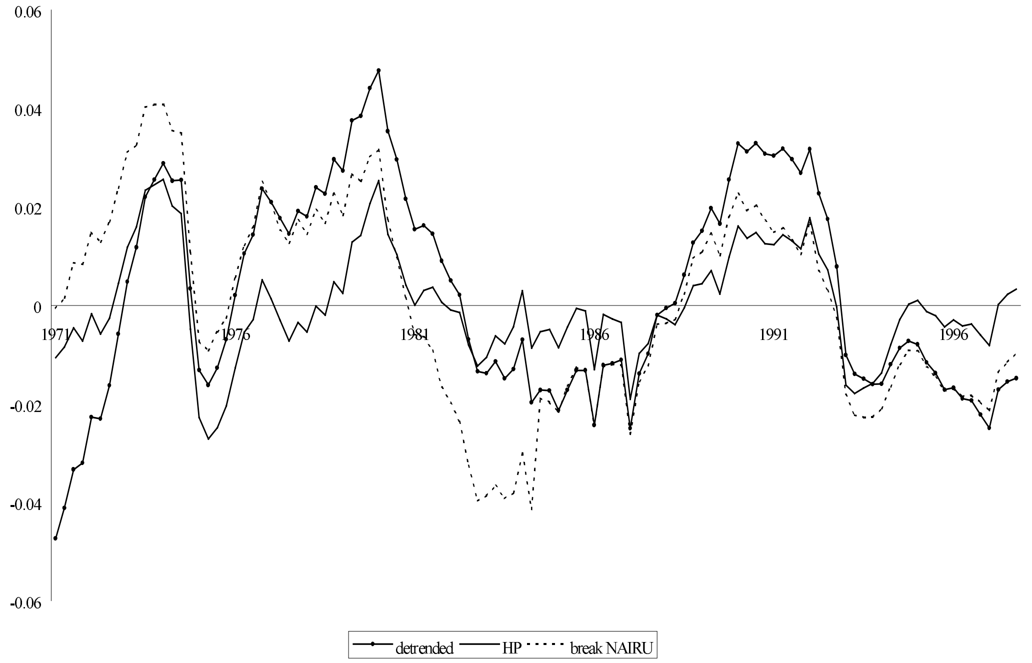
The figure highlights the similar profile of the three series. From close-to-zero output gap in the early seventies, all measures increase until the first energy crisis, rebound again afterwards and dip during most of the eighties. After positive values at the turn of the eighties, they become again negative in the early nineties. Notwithstanding the high number of similarities, some differences stand out in the graph, in particular as concerns the output gap levels at the beginning and at the end of the time horizon. A striking one is the behaviour of the measure derived from the constant NAIRU: it varies from very high levels in the seventies to very low ones in the nineties. The other measures are smoother, with the time varying parameter method giving the more stable measure (in terms of variations around the mean) and the break model lying somewhere in between. This feature is obviously a mechanical consequence of the way the three measures have been derived. There is in fact a systematic bias in all of them, the one characterising the constant NAIRU being the largest, since it clearly overestimates the NAIRU at the beginning of the sample and underestimates it at the end. The interesting point about this fact is that allowing for variations in the NAIRU actually increases the plausibility of the resulting output gaps.

Figure 6.2 compares our measure of the output gap with alternative ones, based on more standard techniques. The three series depicted are output gaps derived from detrended output and a Hodrick-Prescott-filtered output, together with the break-NAIRU one.[40] Although the three measures do not give completely different pictures in terms of output

[40] This measure was chosen simply because it lied between the other two most of the time.

gap patterns, it is evident that they present a higher dispersion than those in the previous graph.

Figure 6.2 Comparison between different measures of the output gap



7 Conclusions

The analysis presented in this paper has investigated whether *the concept* of an area-wide NAIRU and *particular measures* of it make sense and may hence potentially serve as an indicator, among many others, for assessing future price developments arising from pressures in the labour market.

In this light, an exercise aimed at measuring the euro area NAIRU on the basis of a range of time series techniques and on aggregate data has been carried out. The analysis has mainly focused on methodological issues, on the usefulness of such techniques when applied to area-wide data and on the robustness of the resulting estimates with respect to the choice of the model and to different specifications. A special focus has been put in deriving NAIRU measures within the context of Phillips curves, in a broad sense, by estimating a number of alternative specifications embodying different forms of the concept, although other techniques have also been used.

The relative simplicity of the empirical methods adopted may be considered somehow at odds with the complexity of the NAIRU concept itself, which rests often on economic mechanisms that depend on institutional and deep structural factors. However, the concept is broad enough to be treated with simple time-series techniques. Furthermore, an econometric analysis aimed at obtaining meaningful measures of it can be regarded as being "successful" if it provides a core of different estimates that are not inconsistent with each other and have similar properties in terms of relationships with other variables.

The results obtained in this paper point to some interesting features. Different measures of the NAIRU based on Phillips-curve-type relationships seem to be similar and consistent enough to believe that they are capturing the same concept. Moreover, such estimates – stemming both from models in which the NAIRU is treated as a constant and from models in which it is allowed to change over time in a discrete or continuous manner – seem to be robust with respect to alternative specifications and to the choice of the inflation indicator. The obtained measures show significant inflation forecasting ability and yield similar output gap measures, which do not significantly contrast with the evidence provided by alternative methodologies. Clearly, as already mentioned, each of these measures is subject to a systematic bias embedded in the estimation technique adopted, which affects in particular the different level of the various output gaps at the beginning and at the end of the sample period.

Three main conclusions can be drawn, hence, based on the aspects of the results described so far. *Firstly*, the observed unemployment rate seems to be a good leading indicator of inflation. *Secondly*, the simple break model, accounting for shifts in trend during the eighties, seems to perform better than the constant-NAIRU. *Thirdly*, more sophisticated versions of time-varying NAIRUs do not outperform the break model. It is logical to assume that refining the way breaks in trend unemployment are modelled might yield better leading indicators of inflation. The evidence collected in this paper points to a visible change in the inflation-unemployment relationship at the beginning or middle of the eighties. It is highly unlikely that this change took the form of a sudden shift in the NAIRU, as implied by the break model. The failure of the time varying parameter model to account more accurately for this shift is probably a consequence of the complex nature of the latter. A more refined modelling of the NAIRU is hence a research strategy worth pursuing.

Last but not least, it is interesting to briefly recall the aggregation problem discussed in the introduction. *If* the conclusion that there was a shift in the euro area NAIRU is accepted, this implies that there was a general move in this direction all over the countries in the area, or at least in those accounting for the largest share of aggregate unemployment. The alternative view would be that only national unemployment rates matter because they are buffeted by country-specific shocks, and that movements observed at the aggregate level are devoid of information, i.e. random. This paper challenges this view, and proposes an alternative one by which *some* of the shocks affecting the area's labour markets were common across countries. The chosen modelling strategy, on the other hand, precludes any further analysis of the sources of these common shocks, or of their degree of commonality.

References

Apel, M. and P. Jansson (1997), "System Estimates of Potential Output and the NAIRU", *Sveriges Riksbank Working Paper Series* n. 41.

Apel, M. and P. Jansson (1998), "A Theory-Consistent Approach for Estimating Potential Output and the NAIRU", *Sveriges Riksbank Working Paper Series* n. 74.

Ball, L. (1996), "Disinflation and the NAIRU", *NBER Working Paper* n. 5520.

Bank of England (1999), *Economic Models at the Bank of England*, Bank of England.

Baxter M. and R. C. King (1995), "Measuring Business Cycles Approximate Band-Pass Filters for Economic Time Series", *NBER Working Paper* n. 5022.

Beveridge, S. and C. R. Nelson (1981), "A New Approach to Decomposition of Economic Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'", *Journal of Monetary Economics*, 7(2), 151-74.

Bianchi, M. and G. Zoega (1998), "Unemployment Persistence: Does the Size of the Shock Matter?", *Journal of Applied Econometrics*, 13, 283-304.

Blanchard, O. and L. Summers (1986), "Hysteresis and the European Unemployment Problem", *NBER Macroeconomics Annual*, 15-77.

Blanchflower, D. and A. Oswald (1994), *The wage curve*, Cambridge: MIT Press.

Cote', D. and D. Hostland (1994), "Measuring Potential Output and the NAIRU as Unobserved Variables in a Systems Framework", in *Economic Behaviour and Policy Choice under Price Stability*, Bank of Canada, 357-418.

Clark, P., Laxton, D. and D. Rose (1995), "Asymmetry in the U.S. Output-Inflation Nexus: Issues and Evidence", *IMF Working paper* WP/95/76.

Elmeskov, J. (1993), "High and Persistent Unemployment: Assessment of the Problem and its Causes", *OECD Economics Department Working Paper* n. 132.

Estrella, A. and S. Mishkin (1998), "Rethinking the Role of NAIRU in Monetary Policy: Implications of Model Formulation and Uncertainty", *NBER Working Paper* n. 6518.

Fabiani, S., Locarno, A., Oneto, G. and P. Sestito (1997), "NAIRU: Incomes Policy and Inflation", *OECD Economics Department Working Paper* n. 187.

Fagan, G., Henry, J., Mestre, R. (2000), "An Area-Wide Model for the EU11", forthcoming in the *ECB Working Paper Series*.

Giorno, C., Deserres, A. and P. Sturm (1997), "Macroeconomic Policy and the NAIRU in European Economies", *EUI Working Paper* n. 50/97.

Goldfeld, S. M. and R. E. Quandt (1973), "The Estimation of Structural Shifts by Switching Regressions", *Annals of Economic and Social Measurement*, 2(4), 475-85.

Godfrey, L. G. and M. H. Pesaran (1983), "Tests of non-nested regression models: small sample adjustments and Monte Carlo evidence", *Journal of Econometrics*, 21, 133-54.

Goodfriend, M. and R. G. King (1997), "The New Neoclassical Synthesis and the Role of Monetary Policy", *NBER Macroeconomics Annual*.

Gordon, R. (1982), "Price Inertia and Policy Ineffectiveness in the Unites States, 1890-1980", *Journal of Political Economy*, 90, 987-1117.

Gordon, R. (1996), "The Time-Varying NAIRU and its Implications for Economic Policy", *NBER Working Paper* n. 5735.

Gordon, R. (1998), "Foundations of the Goldilocks Economy: Supply Shocks and the Time-Varying NAIRU", *Brookings Papers on Economic Activity*, 0(2), 297-333.

Hansen, B. E. (1994), "Testing for Parameter Instability in Linear Models", in Ericsson, N.R. and J.S. Irons (eds.), *Testing exogeneity*, Oxford and New York, Oxford University Press.

Laubach, T. (1997), "Measuring the NAIRU: Evidence from Seven Economies", *Federal Reserve Bank of Kansas City Working Paper* n. 97-13.

Layard, R., Nickell, S. and R. Jackman (1991), *Unemployment: Macroeconomic performance and the labour market*, Oxford, Oxford University Press.

Laxton, D. and R. Tetlow (1992), "A Simple Multivariate Filter for the Measurement of Potential Output", *Bank of Canada Technical Report* n. 59.

Laxton, D., Meredith, G. and D. Rose (1994), "Asymmetric Effects of Economic Activity on Inflation: Evidence and Policy Implications", *IMF Working Paper* WP/94/139.

Lindbeck, A. and D. Snower (1988), *The Insider-Outsider Theory of Employment and Unemployment*, Cambridge, MA, MIT Press.

Morgan, J. and A. Mourougane (1999), "Wage formation and labour market institutions in Europe", *ECB mimeo*, presented at the ECB Workshop on Structural Unemployment, 13-14 December, 1999.

Rao, C.R. (1973), *Linear statistical inference and its applications*, New York, Wiley and sons.

Rasi, C. and Viikari, J. (1998), "The Time-Varying NAIRU and Potential Output in Finland", *Bank of Finland Discussion Papers* n. 6/98.

Roberts, J. M. (1997), "The Wage Curve and the Phillips Curve", Federal Reserve Board, mimeo.

Staiger, D., Stock, J. and M. Watson (1996), "How Precise are Estimates of the Natural Rate of Unemployment?", *NBER Working Paper* n. 5477.

Staiger, D., Stock, J. and M. Watson (1997), "The NAIRU, Unemployment and Monetary Policy", *Journal of Economic Perspectives*, 11(1), 33-49.

Stock, J. H. (1999), "Monetary Policy in a Changing Economy: Indicators, Rules and the Shift Towards Intangible Output", *IMES Discussion Paper* n. 99-E-13.

Watson, M. W. (1986), "Univariate Detrending Methods with Stochastic Trends", *Journal of Monetary Economics*, 18(1), 49-75.

Whelan, K. (1997), "Wage Curve vs. Phillips Curve: Are There Macroeconomic Implications?", *Federal Reserve Board, mimeo*.

Annex A

Phillips curves are a widespread tool of analysis thanks to their inherent simplicity and the ease with which they can be understood. Notwithstanding that, there are potential pitfalls in their use that should be avoided, particularly as relates to monetary policy. As already explained, the framework underlying the approach followed in this paper is the natural rate hypothesis, which postulates that the real economy is in the long run independent from monetary policy. The way this statement has been embodied in this paper is by assuming *static* and *dynamic* homogeneity, i.e. by postulating equations with some form of nominal zero-homogeneity property in price levels *and* first differences. In a nutshell, this property ensures that in the long run the price system can be expressed in terms of relative prices, no nominal anchor being part of the system. Obviously, this implies that the price system equations are unable to tell much about inflation at any relevant time horizon.

In more formal terms, equations (A.1a) and (A.1b) may describe a (simplified) version of a price system of an economy. The first one is a price equation which links the price level p to a measure of costs labelled as c. This measure may be understood as unit labour costs—i.e., wages minus labour productivity—to remain within the approach followed in the text. Other factors affecting prices and costs are collected in the variables z^p and z^c. These variables are all assumed to be non-trended and independent in the long run from nominal factors. The unemployment gap could form part of both. Lags and leads of the relevant variables enter through the lag-polynomials $A_p(L)$, $A_c(L)$, $B_c(L)$ and $B_p(L)$. Obviously, leads in the equations are to be understood as expectations of the corresponding variables.

$$A_p(L) \cdot p = A_c(L) \cdot c + z^p \qquad \text{(A.1a)}$$

$$B_c(L) \cdot c = B_p(L) \cdot p + z^c \qquad \text{(A.1b)}$$

A reduced-form analogue of the system is expressed in (A.2), where costs have been substituted in (A.1a) by their expression in terms of prices—i.e., by inverting (A.1b).[41] *Static homogeneity* holds if and only if the sum of lags-leads in prices adds up to zero, which means that the equation (A.2) can be re-expressed in terms of inflation ($A(L)$ has a unit root). In turn, *dynamic homogeneity* holds if and only if the sum of lags-leads of

[41] Assuming that p and c are of the same order of integration ensures that $B_c(L)$ is invertible.

inflation in the transformed equation also adds up to one ($A(L)$ has *two* unit roots). In this case, the equation can be re-expressed in terms of accelerations in inflation.[42]

$$A(L)\cdot p = z,$$
$$\text{where } A(L) \equiv \left[A_p(L) - A_c(L)\cdot B_c(L)^{-1}\cdot B_p(L)\right], \qquad \text{(A.2)}$$
$$\text{and where } z \equiv A_c(L)\cdot B_c(L)^{-1}\cdot z^c + z^p$$

In terms of the original system, it is sufficient for static homogeneity to hold that $A_p(1)=A_c(1)$ and $B_c(1)=B_p(1)$, i.e. for the sum of lags and leads in the lag-polynomials of each equation to be equal. Under this assumption, the system can be re-written as in (A.3a) and (A.3b), where the long-run behaviour of the system has been isolated in the lagged term p-c.[43] It is worth pointing out that static homogeneity is a linear restriction that does not affect the order of integration of the variables, but nevertheless the lagged p-c term would become an ECM term should prices and costs be integrated of order 1.

$$A_p^*(L)\cdot\Delta p = A_c^*(L)\cdot\Delta c - A_p(1)\cdot(p-c)_{-1} + z^p \qquad \text{(A.3a)}$$
$$B_c^*(L)\cdot\Delta c = B_p^*(L)\cdot\Delta p - B_c(1)\cdot(c-p)_{-1} + z^c \qquad \text{(A.3b)}$$

Correspondingly, dynamic homogeneity holds in (A.3) if $A_p^*(1) = A_c^*(1)$ and $B_c^*(1) = B_p^*(1)$, in which case the previous operation can be repeated, and the system re-written as in (A.4). Again, the fact that variables enter in second differences is an expression of the linear restrictions assumed, but is not a statement of the stationarity properties of the data.

$$A_p^{**}(L)\cdot\Delta^2 p = A_c^{**}(L)\cdot\Delta^2 c - A_p^*(1)\cdot\Delta(p-c)_{-1} - A_p(1)\cdot(p-c)_{-1} + z^p \qquad \text{(A.4a)}$$
$$B_c^{**}(L)\cdot\Delta^2 c = B_p^{**}(L)\cdot\Delta^2 p - B_c^*(1)\cdot\Delta(c-p)_{-1} - B_c(1)\cdot(c-p)_{-1} + z^c \qquad \text{(A.4b)}$$

One interesting feature of these equations is that the system describes the behaviour of relative prices (in the form of p-c) up to short-term developments. In other words, the system evolves most of the time through the space spanned by relative prices, with occasional and short-lasting departures. These departures are an important element of reduced-form Phillips curve like (A.2): Phillips curves embodying dynamic homogeneity

42 In which case expression (2.2) in the main text applies.

43 Given a non-null lag-polynomial $\theta(L)$, its starred version $\theta^*(L)$ results from adding and subtracting its lagged long-run value, $\theta(L)=[\theta(L)-\theta(1)L]+\theta(1)L$, and noting that the term in squared brackets has a unit root that can be factored out, $\theta(L)=[\theta^*(L)(1-L)]+\theta(1)L$. The long-run terms of the variables can then be grouped due to the assumption that they all have the same value.

cannot be used to analyse medium- and long-term inflation developments. To see why, notice first that the presence of *p-c* in levels in both equations implies that in steady-state both variables will grow at the same speed (which we will assume stable). Otherwise, an ever-widening gap would appear between the two, for which no variable in the equation could account. This means that $\Delta(p\text{-}c)$, $\Delta^2 p$ and $\Delta^2 c$ will all be zero in the steady-state, implying that the final level of the real cost (real wage devoid of productivity, in this case) will be a linear combination of the variables z^p or z^c, and will thus not depend on nominal factors. Thus, although unemployment or the unemployment gap may prove in this context to be a good indicator of inflationary pressures, it cannot be treated as a variable determining the future path of nominal variables.

European Central Bank Working Paper Series

1 "A global hazard index for the world foreign exchange markets" by V. Brousseau and F. Scacciavillani, May 1999.

2 "What does the single monetary policy do? A SVAR benchmark for the European Central Bank" by C. Monticelli and O. Tristani, May 1999.

3 "Fiscal policy effectiveness and neutrality results in a non-Ricardian world" by C. Detken, May 1999.

4 "From the ERM to the euro: new evidence on economic and policy convergence among EU countries" by I. Angeloni and L. Dedola, May 1999.

5 "Core inflation: a review of some conceptual issues" by M. Wynne, May 1999.

6 "The demand for M3 in the euro area" by G. Coenen and J.-L. Vega, September 1999.

7 "A cross-country comparison of market structures in European banking" by O. de Bandt and E. P. Davis, September 1999.

8 "Inflation zone targeting" by A. Orphanides and V. Wieland, October 1999.

9 "Asymptotic confidence bands for the estimated autocovariance and autocorrelation functions of vector autoregressive models" by G. Coenen, January 2000.

10 "On the effectiveness of sterilized foreign exchange intervention" by R. Fatum, February 2000.

11 "Is the yield curve a useful information variable for the Eurosystem?" by J. M. Berk and P. van Bergeijk, February 2000.

12 "Indicator variables for optimal policy" by L. E. O. Svensson and M. Woodford, February 2000.

13 "Monetary policy with uncertain parameters" by U. Söderström, February 2000.

14 "Assessing nominal income rules for monetary policy with model and data uncertainty" by G. D. Rudebusch, February 2000.

15 "The quest for prosperity without inflation" by Athanasios Orphanides, March 2000

16 "Estimating the implied distribution of the future short term interest rate using the Longstaff-Schwartz model" by Peter Hördahl, March 2000

17 "Alternative measures of the NAIRU in the euro area: estimates and assessment" by Silvia Fabiani and Ricardo Mestre, March 2000