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RICARDO MESTRE

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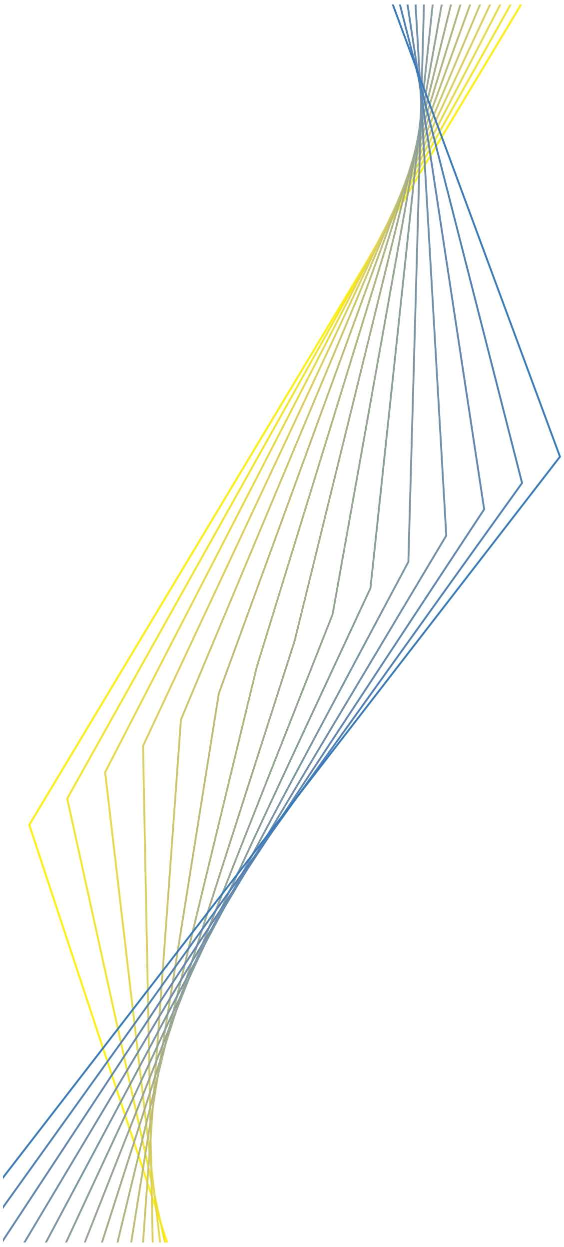

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Abstract

This paper addresses the issue of measuring the NAIRU for the euro area and assessing the robustness and precision of the obtained estimates. The empirical framework adopted is based on systems combining an Okun-type relationship between cyclical unemployment and the output gap with a Phillips curve and stochastic laws of motion for the NAIRU and potential output. Such systems have been estimated using Kalman-filter techniques.

The aim of this approach is double: on the one hand, recent advances in the mentioned techniques are exploited, also with the aim of assessing the degree of uncertainty around the derived measures; on the other hand, the robustness of the approach is tested by looking at alternative versions of the systems themselves.

The results obtained point to an estimate of the area-wide NAIRU that is robust to changes in the underlying models. This robustness is shown to hold both in terms of the mean – i.e., the shape of the resulting NAIRU – and the variance of the process. The latter is derived through bootstrap exercises using the models alone or pooled together. The evidence found suggests that the increase in the aggregate NAIRU that took place in the early part of the sample period has come to a halt and may be about to be reversed. The estimated final level for the NAIRU is around 10 per cent. Furthermore, the bootstrap exercises point to confidence bands close to 1 per cent around the estimated value.

Non-Technical Summary

Interest in the Phillips curve and the unemployment-inflation link has recently increased. The literature has found a number of interesting facts concerning the links between inflation and demand pressures measured in terms of cyclical variations in unemployment. The empirical evidence highlights the statistical robustness of this link, coupled with a large degree of uncertainty surrounding the implicit NAIRU, the level of unemployment that leaves inflation unchanged.

Although much scarcer than for the US, several studies have focused also on European countries, adopting technical approaches broadly similar to those used for the US. One interesting empirical framework, on which the present paper is based, was initiated by the work of Apel and Jansson (1999, 1999a and 1999b), which estimates relatively complex time-series-based systems. Such systems combine a relationship between cyclical unemployment and the output gap (the so-called Okun law) with a Phillips curve and a description of the data-generating processes driving both the NAIRU and potential output.

This paper attempts to contribute to and extend the analysis of an euro area-wide measure of the NAIRU, using time-series techniques rather than more structural approaches, with the aim of searching for plausible measures of the aggregate NAIRU and assessing their robustness and precision. In so doing, we investigate what type of models provide a reasonable NAIRU profile, and whether a range of simple models sharing a number of features tell, broadly speaking, the same story about the evolution of this unobservable variable. In addition, we exploit these models to assess the degree of uncertainty around the derived measures.

The results obtained point to area-wide NAIRU estimates that are robust to changes in the underlying models, as long as the models belong to a specific class that is described in the text. This robustness is shown to hold both in terms of the shape of the resulting NAIRU and its variability over time. The approach also provides an interesting insight into the past behaviour of unemployment and inflation in Europe. It also provides some evidence that labour market reforms might be starting to have a positive effect at the macroeonomic level in the euro area as a whole.

A System Approach for Measuring the Euro Area NAIRU

Silvia Fabiani and Ricardo Mestre*

1. Introduction

Interest in the Phillips curve and the unemployment-inflation link has recently and strongly increased in the US. The rebirth of the concept can be exemplified by evidence gathered in Gordon (1996 and 1998), Staiger *et al.* (1996, 1997), Stock and Watson (1999) or, in a less technical but appealing presentation, in Fuhrer (1995). This literature has found a number of interesting facts concerning the links between inflation and demand pressures measured in terms of cyclical variations in unemployment. Notable evidence is the statistical robustness of this link in the US (see Fuhrer, *op. cit.*), coupled with a large degree of uncertainty surrounding the implicit cycle-free NAIRU (see Staiger *et al.*, *op. cit.*). Last but not least, unemployment has been found to be a reliable leading indicator of inflation (see Stock and Watson, *op. cit.*).[1]

Related analysis for the euro area as a whole is much scarcer, a significant exception being Orlandi and Pichelman (2000). However, several studies have focused on European countries, adopting time-series techniques in line with those used in the above mentioned literature (see for example Laubach, 1997; Boone and Turner, 1999; OECD, 2000; Boone, 2000). One interesting empirical approach, on which the present paper is based, was initiated by the work of Apel and Jansson (1999, 1999a and 1999b), in which relatively more complex systems than the ones adopted in the above mentioned studies are estimated. Such systems combine an Okun-type relationship between cyclical unemployment and the output gap with a Phillips curve and stochastic laws of motion for the NAIRU and potential output. This approach, by expanding the structural information available to the analyst, has the important advantage of enabling the estimation of all the relevant parameters of the system. Conversely, the standard approach, which is generally based on a single cyclical indicator – the unemployment gap – and on the assumption that the NAIRU moves as a pure random walk, leads to non-identification of the variance of

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[1] For a more detailed review of the empirical literature on the issue of the estimation of the NAIRU, see Fabiani and Mestre (2000).

the process (see Gordon, *op. cit.*). Also modelling the short-term fluctuations of a single cyclical indicator (see Gerlach et al. (1999) for an example on the output gap) allows the determination of all the relevant parameters, but, overall, the use of more than one indicator has been found to considerably narrow down the uncertainty with which parameters are estimated.

This paper will attempt to contribute to and extend the analysis of an euro area measure of the NAIRU, using time-series-based techniques rather than more structural approaches, with the aim of searching for plausible measures of the NAIRU and assessing their robustness and precision. In so doing, the authors will investigate what type of models provide a reasonable NAIRU profile, and whether a range of simple models sharing a number of features tell, broadly speaking, the same story about the evolution of this aggregate unobservable variable. Last but not least, the authors will exploit these models to assess the degree of uncertainty around the derived measures.

It is well known that the concept of the NAIRU presents a number of problems both at the conceptual level and at the empirical level. This paper does not deal with conceptual problems (see Fabiani and Mestre (2000) and Mac Adam and Mc Morrow (1999) for a discussion), it focuses instead on the empirical side and in extensively discussing issues found in the derivation of the NAIRU for the euro area. As already announced, the only tool considered are reduced-form, time-series-based methods delivering sensible smoothed versions of observed unemployment – which will be termed as NAIRU – able to give a reasonable account of historically observed dynamic patterns of unemployment. Our description of the NAIRU will hence embody structural factors and influences in an “all-encompassing” measure.

Necessarily, the approach is descriptive rather than explanatory. As a consequence, the models tested have been chosen with respect to their in-sample properties and the degree to which they were able to match the behaviour of the series: it has not been our primary goal to estimate an accurate forecasting tool. This goal has lifted a number of constraints that could have plagued the analysis. First, contrary to widespread practice, we have focused from the start on specifications of the Phillips curve embodying dynamic homogeneity and thus disentangling the NAIRU from nominal factors. Second, we have explored a range of laws of motion for the NAIRU in order to informally optimise the plausibility of the resulting estimates but with no regard for their forecasting properties. These two aspects lead to an estimate of the NAIRU that is in principle independent from secular changes in the inflation rate but not necessarily easy to project. In other words, we have used the past to describe the present, rather than to project it into the future.

The rest of the paper is structured along the following lines. Section 2 describes the models chosen to derive the range of estimates for the NAIRU. Section 3 briefly introduces the dataset used in the analysis and presents the estimates themselves. It also highlights a number of points regarding them. Section 4 tries to assess the degree of uncertainty in the measurement of the NAIRU using the estimated models for simulation analysis. The last part of the paper includes a summary of conclusions, lessons and remaining issues.

2. The system

The suite of models that have been used are essentially based on a Phillips curve relationship, an Okun law and a set of equations defining the law of motion of the unobservable variables included in the system, namely potential output and the NAIRU.

The Phillips curve provides a link between inflation (first differences of prices or wages), the state of aggregate demand (either the unemployment or the output gap) and a vector of supply-side shocks intended to capture shifts in the inflationary pattern not related to the demand side. Such factors might include, for example, import prices, energy prices, productivity or unit labour costs. Inflation is assumed to depend only on nominal factors in the long run. (In other words, the coefficients of lagged inflation are constrained to add up to unity.)[2] This long-run nominal homogeneity restriction allows expressing the Phillips curve in terms of first differences of inflation and guarantees the existence of an equilibrium value for unemployment or output. Such equilibrium variables are treated as endogenous and estimated simultaneously with the parameters of the model as unobservable variables.

The Okun law captures the relationship between the output gap and the unemployment gap, the direction of the causality varying according with the assumptions concerning the most relevant sources of shocks affecting the system.

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[2] See Fabiani and Mestre (2000), Annex A, for an analytical description of the implications of the assumption of long-run nominal homogeneity in the price equation.

As for the equations defining the law of motion for the unobservable variables, the assumption we make in this work is that both the NAIRU and potential output follow a random walk process supplemented with a variable stochastic drift, a process denoted as 'local linear trend' in the literature.

The choice for the law of motion of the NAIRU is not widespread in the literature and deserves some background discussion. One important aspect of early stages in the empirical analysis of the paper is that a proper modelling of the NAIRU along the proposed lines does not lead to reasonable results unless smooth changes in the trended behaviour of the variable are allowed for. Preliminary analysis done by the authors but not reported in the text shows in fact that, contrary to the US case, in which assuming a stationary level for the variable is appropriate, estimates of the euro area NAIRU under this assumption are highly implausible. Instead, a smooth stochastic trend is needed to increase the plausibility of results. Although this seemingly implies that unemployment is an I(2) variable, it is our view that this trend reflects an important upward shift that took place somewhere along the route (probably in the 70's and/or 80's) but no longer present, or even about to be reversed. In other words, we consider that the sample used incorporates movements of the NAIRU that cannot be captured by models embodying a single unit root for the variable, but that as new observations will be added to the sample a more stationary behaviour may emerge. Thus, this stochastic trend might be capturing sample-specific features but (hopefully) not structural features of the current non-observed *normal* level of unemployment.[3]

2.1 The baseline system

In the baseline specification of the model broadly described above, inflation is measured on the basis of the consumption deflator, the stance of aggregate demand is specified in terms of the unemployment gap and the causality in the Okun law runs from cyclical fluctuations in unemployment to output. The variables are defined as follows: π is the inflation rate (first difference of the log of consumption deflator), z is a vector of supply-side variables affecting inflationary pressures, u is the unemployment rate, y is the (log

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[3] Obviously, this point cannot be proved with the current sample and is thus not part of the conclusions drawn in the paper. The presence of a second unit root for the NAIRU has to be accepted on purely empirical grounds, but this should not prevent a reading of the evolution of the estimated NAIRU in the light of potential sample-dependent distortions due to a structural shift in an otherwise relatively stable process. In this, the authors make an interpretation of the results—and present it as such—rather than an exposition of evidence found.

of) output level, u* and y* represent the NAIRU and (the log of) potential output, respectively. The system is composed of the following seven equations:

$$\Delta\pi_t = \alpha + a(L)\Delta\pi_{t-1} + \rho(L)(u_{t-1} - u_{t-1}^*) + b(L)z_t + \varepsilon_t^{\pi} \tag{2.1}$$

$$y_t - y_t^* = \phi(L)(u_{t-1} - u_{t-1}^*) + \varepsilon_t^{yc} \tag{2.2}$$

Potential output and the NAIRU are assumed to follow a so-called local linear trend model:

$$y_t^* = y_{t-1}^* + \beta_{t-1} + \varepsilon_t^{y^*} \tag{2.3}$$

$$u_t^* = u_{t-1}^* + \xi_{t-1} + \varepsilon_t^{u^*} \tag{2.4}$$

where the two stochastic trends β and ξ are defined as:

$$\beta_t = \beta_{t-1} + \varepsilon_t^{\beta} \tag{2.5}$$

$$\xi_t = \xi_{t-1} + \varepsilon_t^{\xi} \tag{2.6}$$

The unemployment gap is modelled as an autoregressive process:

$$u_t - u_t^* = \delta(L)(u_{t-1} - u_{t-1}^*) + \varepsilon_t^{uc} \tag{2.7}$$

All error terms in equations (2.1) to (2.7) are assumed to be IID with zero mean and constant variance and to be mutually uncorrelated.

In order to express this system in its state-space form for the Kalman filter estimation, (2.1) is used to derive the measurement equation (together with two identities defining output and unemployment as the sum of their trend and cyclical components) and equations (2.2) to (2.7) are used to derive the transition equation.[4] All the relevant parameters – the coefficients of equations (2.1) to (2.7) and the variances of the error terms – and the two unobservable variables can be estimated by maximum likelihood.

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[4] For a more formal description of the model in its state-space form, see Appendix A.

The possibility of estimating by an optimisation algorithm all the variance parameters of the system is an aspect of particular relevance. A majority of empirical models developed in the literature to estimate a time-varying NAIRU by means of Kalman filters tend in fact to specify the transition equation only in terms of NAIRU, without allowing for a link between the latter and potential output in terms of an Okun law. One of the consequences of adopting such a simple structure is that the optimisation procedure often produces estimates of the error variance of the NAIRU that tend to absorb all the residual variation of the Phillips curve (see for example Gordon, 1996 and Staiger at al, 1996). As a solution, most studies on the subject resort to fixing the signal-to-noise ratio (i.e. the ratio of the variance of the residuals of the transition and the measurement equations) exogenously, so that the estimated NAIRU is relatively smooth (see Gordon, 1996). As the results of our exercise – discussed in the next section – will confirm, this problem does not seem to arise in more structured models such as the one presented in this paper (see also Apel and Jansson, 1999, 1999a, 1999b). However, it is true that the simple addition of an assumption on the dynamics of cyclical unemployment allows the estimation of a time-varying NAIRU without the need of fixing the signal to noise ratio (as was done by Gerlach et al. (1999) using the output gap).[5]

2.2 Alternative system specifications

In order to investigate how sensitive the results arising from the described model are to changes in specification, we modify the latter in a number of different ways, of which we present in what follows only the ones we consider as most relevant. All the alternative models should be understood as 'local' alternatives to the baseline model, in that simple links contained in the original model are slightly changed in ways described below. All the models revolve around the same idea of describing relationships between inflation, the unemployment gap and the output gap using different versions of a Phillips curve and an Okun law. The interesting feature of the exercise is the non-encompassing nature of the models, as this leads to a form of robustness test of the baseline model: there is no general-to-specific simplification leading to better-behaved models. Far from that, models are different along specific dimensions and should lead to different estimates of the NAIRU. Whether these estimates are fundamentally different or rather similar is an empirical issue to be dealt with.

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[5] We estimated this simple model, which we define as "extended time-varying-parameter model", but the results were considerably less satisfactory than those reported in the text. We report such results in Appendix B.

Model 1 – In the first alternative specification, we investigate the possibility that the output gap might be a better indicator of the economic cycle than the difference between observed unemployment and the NAIRU. In this respect, the variable measuring the stance of aggregate demand in the Phillips curve is expressed in terms of output rather than unemployment gap. Also the causality in the Okun law is reversed with respect to the baseline, running from output to unemployment. Therefore, the baseline system is modified so that equations (2.1), (2.2) and (2.7) are replaced by:

$$\Delta\pi_t = \alpha + a(L)\Delta\pi_{t-1} + \rho(L)(y_{t-1} - y^*_{t-1}) + b(L)z_t + \varepsilon_t^{\pi} \tag{2.1a}$$

$$u_t - u_t^* = \phi(L)(y_{t-1} - y^*_{t-1}) + \varepsilon_t^{uc} \tag{2.2a}$$

$$y_t - y_t^* = \delta(L)(y_{t-1} - y^*_{t-1}) + \varepsilon_t^{yc} \tag{2.7a}$$

This modification implies a redefinition of the transition and the measurement matrices in the state-space form of the system, all other elements remaining unchanged.

Model 2 – The baseline model and what we labelled above as model 1 share, among others, a particular common feature: in the relationship linking output and unemployment cyclical movements, the causality necessarily runs from either of the two variables to the other. In other words, these models do not allow for the presence of shocks that might affect contemporaneously both output and unemployment fluctuations. The aim of the exercise described here is to explore this possibility. More in detail, the Okun law, expressed by equation (2.2) in the baseline is replaced by:

$$y_t - y_t^* = \phi(L)(u_t - u_t^*) + \varepsilon_t^{yc} \tag{2.2b}$$

This implies a slightly more complex modification of the baseline model than the one applied above, in that both the transition and the measurement system have to be altered to allow for the presence of a simultaneous relationship in the Okun law. Namely, in the state-space form of this model equations (2.1) and (2.2b) are used to define the measurement system (together with an identity defining unemployment as the sum of its cyclical and trend component), while the transition system is defined on the basis of equations (2.3) to (2.7).

Model 3 - The third specification we consider entails simply a reversion, with respect to the baseline, in the causality underlying the Okun law. Such a reversion, however, implies a higher complexity in the links between unemployment and output than those implicit in the models considered so far. Indeed, whereas the latter assume a passive role for either the output gap or the unemployment gap in explaining inflation, model 3 includes an explicit interaction between the two variables. Namely, we redefine the system so that equations (2.2) and (2.7) are replaced by:

$$u_t - u_t^* = \phi(L)(y_{t-1} - y_{t-1}^*) + \varepsilon_t^{uc} \tag{2.2c}$$

and

$$y_t - y_t^* = \delta(L)(y_{t-1} - y_{t-1}^*) + \varepsilon_t^{yc} \tag{2.7c}$$

This modification requires a redefinition of the transition matrix in the state-space form of the system, all other elements remaining unchanged

Model 4 - As a final exercise, we model the Phillips curve in terms of wage rather than price changes. In particular, we follow Gordon (1999) and adopt as a dependent variable in the Phillips curve the rate of growth of trend unit labour costs, i.e. the rate of growth of nominal wages minus the trend rate of growth of labour productivity. Among the regressors, we include lagged nominal wage growth and contemporaneous and lagged price inflation in order to capture both the process of expectation formation and inertial effects. The variable measuring the stance of aggregate demand in the Phillips curve is the unemployment gap. A long-run dynamic homogeneity restriction on the coefficients of wage and price changes ensures the existence of an equilibrium value for the NAIRU. The vector of supply-side variables includes, as in the baseline model, contemporaneous and lagged values of the second difference of import prices. The causality in the Okun law runs, as in the baseline, from unemployment to output cyclical fluctuations. Therefore, the baseline system is modified so that equation (2.1) is replaced by:

$$\Delta^2\tilde{w}_t = \alpha + a(L)\Delta^2 w_{t-1} + c(L)\Delta^2 p_t + \beta\left(\Delta\tilde{w}_{t-4} - \Delta p_{t-4}\right) + \rho(L)(u_{t-1} - u_{t-1}^*) + b(L)z_t + \varepsilon_t^{\pi} \tag{2.1d}$$

where w is the (log of) nominal wage, $\Delta\tilde{w}_t = \Delta w_t - \Delta prod^*$, $\Delta prod^*$ is the rate of growth of trend labour productivity and the long-run nominal homogeneity has been imposed.[6] This changes the measurement equation in the state-space form of the system, while all other elements of the transition system remain unchanged.

One notable feature of this system is the partial-equilibrium aspect of the equations involved. Although the goal is to model the real wage rate – for which an equation is enough – short-term changes in price inflation different from changes in wage inflation are permitted. The obvious implication of this is that one further equation is needed to explain this difference, for instance by adding a conveniently defined price-wage equation (as opposed to the wage-price equation). The known problems of lack of identification of these systems led us to adopt the partial approach described – fairly standard in the literature, anyway.

3. Estimation and empirical results

This section reports and discusses the estimation results from the models described above, applied to quarterly aggregate data for the euro area over the period 1970Q1-1999Q3.[7] In particular, the series utilised in the applied exercise are the unemployment rate, real GDP, consumption and import deflators and nominal wages.

The five systems were estimated by maximum likelihood using the Kalman filter. This method allows estimating the parameters of the measurement and transition equations, including the error variances of the unobserved variables and the state variables, namely, potential output, the NAIRU and their stochastic trends.

Prior to the system estimation, a preliminary specification search based on OLS was carried out in order to approach the most satisfying specification for each equation composing the system. Since OLS could not be used to derive the unobservable variables, the time series for the latter – the NAIRU and potential output - were estimated on the basis of standard Hodrick-Prescott filters.[8] These two series constitute in fact a necessary information for determining the specification of both the Phillips curve and the dynamic process driving cyclical fluctuations.

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[6] Only real wages (devoid of productivity) enter in first-difference form, which means that a push to one of the nominal variables is translated one-to-one to the other in the long run. In order to impose static homogeneity, test were also carried out including the *level* of the real wage (less productivity) as an additional regressor, but results reported in the text were not significantly affected by this change.

[7] The data are derived from the ECB area-wide model database. For details, see Fagan, Henry and Mestre (2001).

[8] Results were not very sensitive to the value chosen for the smoothing parameter. The standard value of 1600 was finally retained.

As concerns the specification for the Phillips curve, various initial alternatives were investigated. We first focused on whether the demand variable (the unemployment gap or the output gap, depending on the model chosen) entered in first differences in addition to (or rather than in) levels, hence signalling strong persistence or hysteresis in the effects of shocks.[9] The preliminary evidence suggested by OLS results did not seem to strongly support the presence of such effects. Moreover, dropping this term considerably simplified the estimated Kalman-filter system and led to more robust and plausible estimates. Hence, for both the baseline and the alternative models, we decided to adopt a "textbook" specification for the curve describing inflation dynamics, including the demand variable only in levels. The search for exogenous supply-side factors entering the equation was also conducted via OLS estimation. The results showed that it is not easy to find variables that explain aggregate movements in inflation in the whole euro area over the entire estimation period. The only significant factor turned out to be the second difference of import prices.[10] Lags for variables entering the equation – other than the state variable — were conventionally set at four.

As for the Okun law, we tested the significance of the first two lags of the explanatory variable directly in the Kalman-filter estimation. In the models in which the causal link runs from the unemployment gap to the output gap – i.e. the baseline and model 4 – we found that the first lag of the unemployment gap accounted for most of the variation in the output gap. When the causal relationship was reversed – i.e. specifications 1 and 3 – results pointed to a longer lag structure, forcing the inclusion of the second lag of the explanatory variable.

In the equation describing cyclical fluctuations, either in terms of unemployment or output gap, four lags of the autoregressive coefficients were considered. In all models only the first two lags turned out to be significant, their sum being always slightly less than one.

As for the initial values of the parameters to utilise as input for the optimisation algorithm, we adopted the OLS estimates of the single equations composing the system.

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[9] On the evidence of hysteresis in unemployment patterns across European countries and its consequences as far as policy is concerned, see Giorno et al (1997).

[10] For the baseline model, we have also considered an alternative specification of the Phillips curve in which inflation is measured in terms of GDP deflator rather than consumption deflator and the growth rate of unit labour costs enters as an additional exogenous supply side variable influencing inflationary dynamics. Results concerning the relevant parameters and the estimated unobserved components do not seem to vary much with respect to the baseline results presented in Table 1.

Although as a general rule these initial values should not be too far away from the true ones, otherwise the optimisation process might fail to converge, we found our systems to be quite robust to the choice of the starting values.

The results for the baseline model and its four modified versions described above are presented in Table 1. The Table reports estimates for the key parameters, tests on both the individual parameters and the equations as a whole, and the estimated value of the unobserved component identified as the NAIRU at the end of the sample period.[11]

In all models the first lag of the aggregate demand variable appears to be statistically significant in explaining changes in the rate of inflation. The value of the coefficient ρ - with the exception of model 1 in which the Phillips curve is specified in terms of output rather than unemployment gap - is quite low and not very volatile across the various specifications, although in models 3 and 4 it appears to be slightly higher than in the other models. Its sign is consistent with that predicted by theory.

The Ljung-Box Q test for residual autocorrelation and the Doornik-Hansen normality test are satisfactory, their results providing evidence that doesn't allow to reject the null hypothesis of uncorrelated and normally distributed residuals of the estimated Phillips curves.

The estimates for the NAIRU at the end of the sample period (last row of table 1) do not seem to vary widely across the different specifications of the model. This is true also for the model in which the Phillips curve is specified in terms of wage rather than price growth (model 4) and the one in which the unemployment gap in the Phillips curve is replaced by the output gap (model 1). The lowest value for the estimated NAIRU at the end of 1999 is provided by model 4.

As far as the time pattern for the estimated NAIRU is concerned, our estimation provides evidence of a consistent and hence quite robust behaviour of this unobservable variable across the five models considered, as figure 1 clearly shows.[12]

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[11] The table reports also the 5% and 95% confidence bands around the point estimates of the parameters, which were obtained by bootstrapping as described in more detail in the next section.

[12] As we already explained, in the models considered potential output is introduced with the only aim of better pinning down the parameters of the system. Its estimated value for the baseline model is showed in Appendix C.

Table 1 - System estimates

Parameters	*Baseline*			*Model 1*			*Model 2*			*Model 3*			*Model 4*		
Phillips Curve		*5%*	*95%*		*5%*	*95%*		*5%*	*95%*		*5%*	*95%*		*5%*	*95%*
ρ	-0.071	-0.116	-0.040	0.032	0.017	0.050	-0.063	-0.112	-0.030	-0.100	-0.101	-0.093	-0.194	-0.307	-0.082
a_1	-0.505	-0.651	-0.363	-0.513	-0.655	-0.363	-0.494	-0.638	-0.346	-0.523	-0.552	-0.473	-1.242	-1.352	-1.090
a_2	-0.238	-0.380	-0.079	-0.251	-0.396	-0.082	-0.233	-0.375	-0.071	-0.256	-0.258	-0.232	-1.153	-1.308	-0.879
a_3	-0.183	-0.318	-0.045	-0.194	-0.326	-0.049	-0.172	-0.298	-0.036	-0.207	-0.208	-0.187	-1.248	-1.424	-0.913
b_1	0.077	0.056	0.096	0.076	0.055	0.095	0.078	0.057	0.097	0.077	0.069	0.081			
b_2	0.061	0.041	0.083	0.061	0.041	0.083	0.062	0.042	0.083	0.063	0.056	0.064	0.050	-0.014	0.107
b_3	0.036	0.014	0.056	0.037	0.014	0.056	0.037	0.015	0.057	0.038	0.034	0.038			
b_4	0.075	0.051	0.095	0.075	0.051	0.095	0.074	0.051	0.094	0.077	0.070	0.080			
c_1													0.574	0.259	0.883
c_2													0.713	0.323	1.075
c_3													0.778	0.358	1.182
c_4													1.055	0.597	1.401
d_4													-0.835	-1.041	-0.416
Constant	0.000	-0.001	0.000	0.000	-0.001	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	-0.001	-0.002	0.000
Autocorrelation(1)	*7.789*	*(0.65)*		*8.509*	*(0.58)*		*8.012*	*(0.63)*		*9.236*	*(0.51)*		*12.20*	*(0.27)*	
Normality(2)	*1.469*	*(0.48)*		*1.429*	*(0.49)*		*1.778*	*(0.41)*		*1.334*	*(0.51)*		*1.906*	*(0.39)*	
Okun Law															
φ_1	-1.986	-2.084	-1.787	-0.318	-0.364	-0.289	-2.451	-2.759	-2.308	-0.344	-0.353	-0.327	-1.944	-2.060	-1.775
φ_2				-0.063	-0.067	-0.028				-0.063	-0.064	-0.059			
Cyclical Fluctuations															
δ_1	1.876	1.844	1.885	1.822	1.776	1.831	1.770	1.734	1.777	1.724	1.714	1.740	1.877	1.859	1.886
δ_2	-0.898	-0.907	-0.860	-0.848	-0.855	-0.805	-0.791	-0.799	-0.762	-0.753	-0.769	-0.740	-0.895	-0.905	-0.876
Standard Deviations															
σ^{π}	0.00199	0.00251	0.00310	0.00197	0.00250	0.00309	0.00199	0.00252	0.00310	0.00197	0.00255	0.00316	0.00562	0.0069	0.0085
σ^{yc}	0.00000	0.00000	0.00000	0.00202	0.00197	0.00218	0.00000	0.00000	0.00000	0.00210	0.00198	0.00219	0.00000	0.0000	0.0000
σ^{uc}	0.00063	0.00060	0.00074	0.00000	0.00000	0.00000	0.00091	0.00085	0.00098	0.00034	0.00032	0.00036	0.00064	0.0006	0.0007
σ^{y*}	0.00546	0.00545	0.00550	0.00485	0.00477	0.00489	0.00487	0.00477	0.00491	0.00492	0.00467	0.00500	0.00549	0.0055	0.0055
σ^{u*}	0.00105	0.00100	0.00106	0.00100	0.00096	0.00101	0.00085	0.00082	0.00088	0.00078	0.00073	0.00080	0.00105	0.0010	0.0011
σ^{β}	0.00024	0.00006	0.00034	0.00028	0.00000	0.00038	0.00030	0.00000	0.00040	0.00027	0.00025	0.00028	0.00028	0.0001	0.0004
σ^{ξ}	0.00011	0.00001	0.00016	0.00012	0.00008	0.00021	0.00011	0.00003	0.00022	0.00023	0.00021	0.00023	0.00013	0.0000	0.0002
Log-Likelihood	1448.5			1458.14			1485.6			1457.96			1323.99		
NAIRU in Q3 '99	10.59%	10.06%	11.04%	10.39%	9.86%	10.80%	10.52%	9.99%	10.95%	10.27%	9.88%	10.49%	10.21%	9.69%	10.76%

Notes: (1) Ljung-Box Q test measuring general AR(10) residual autocorrelation (p-values in parentheses). (2) Doornik-Hansen normality test (p-values in parentheses).

Figure 1 - Estimated NAIRU from different models

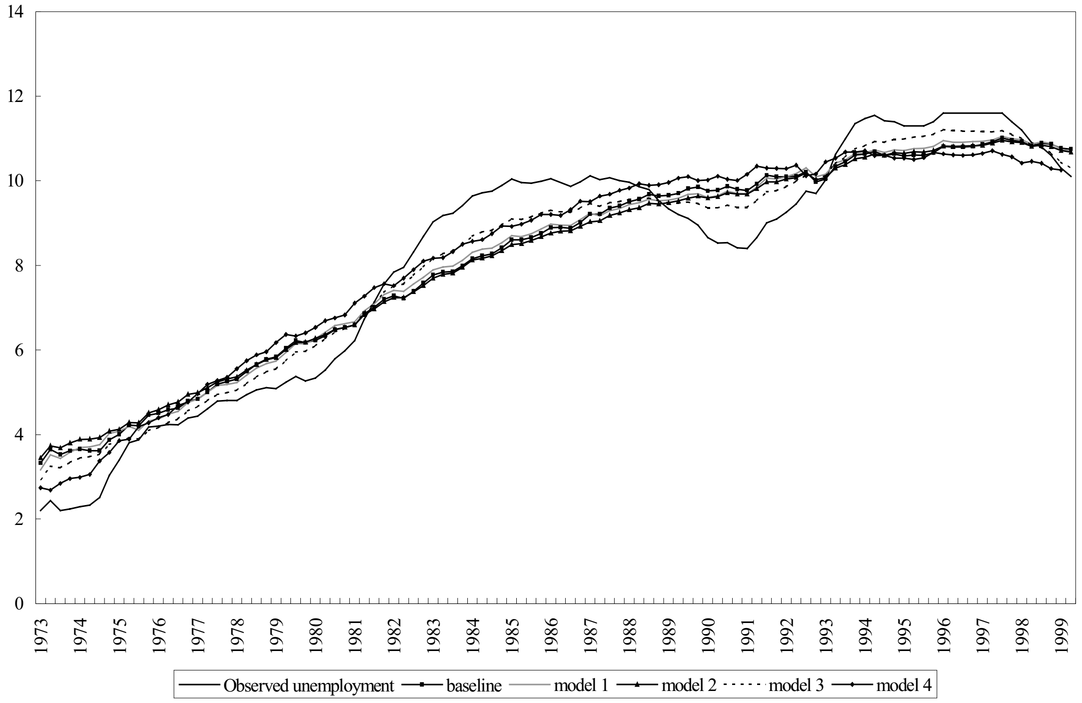
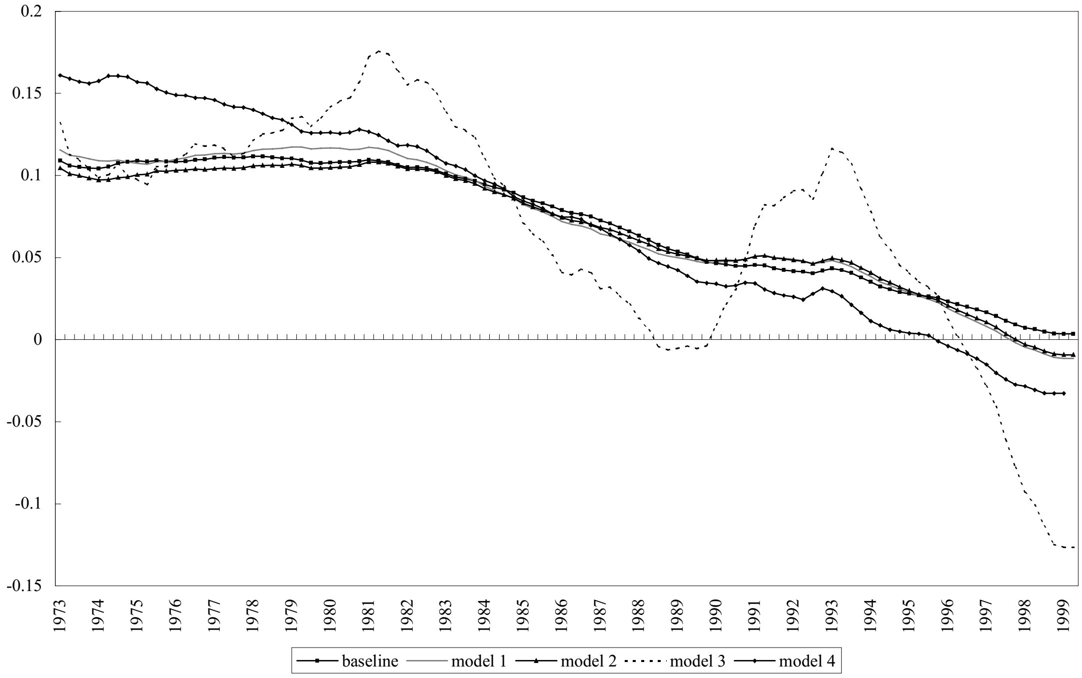


Figure 2 - Estimated stochastic trend in the NAIRU for different models



The trend affecting the NAIRU is also a relevant piece of information that should not be overlooked. Figure 2 collects information on the estimated trends in the NAIRU for all the models involved. It clearly shows a persistent downward pattern, reflecting the smooth deceleration in the NAIRU evident in figure 1. The interesting point about the figure is the apparent change in the behaviour of the trend for systems based on prices, probably the most reliable ones.[13] Although it is not prudent to over-stretch interpretations of the figure, there are signs of a fall in the trend rate of growth of the NAIRU at around the beginning of the eighties. This could imply that the causes underlying the deterioration of observed unemployment in Europe in the 70's and 80's were no longer at work at the end of the period. As for wage-based systems, the interpretation of the trend is slightly more blurred but broadly coherent with the price systems.

4. Measures of uncertainty of the estimated NAIRUs

One of the most important difficulties in the use of Kalman filters is the derivation of measures of uncertainty of the estimated state variables, a point worth taking seriously due to a number of reasons. Firstly, because unless some measure of the uncertainty around the estimated NAIRU is derived, it is difficult to draw firm conclusions from the exercise. Also, because hinting at the correct bounds for the NAIRU has important practical implications for the analysis of the labour market in general, and of its current situation in particular. In this respect, it is important to derive confidence bands comprehensive enough to contain relevant information for the analyst.

As is well known, deriving bands around state variables in a Kalman filter estimation is far from being trivial. This fact stems from the many sources of uncertainty, which include uncertainty on the initial conditions, the non-observability of the state variables and the sample dependence of the estimated hyper-parameters. Furthermore, the relative flatness of the likelihood at estimated parameters for some models, plus the presence of non-stationary behaviour in the systems compound the problem of deriving bands for the NAIRU.

It is obviously difficult to really address all these points with local tests. An alternative procedure is to run comprehensive and extensive simulation exercises and report explicit

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[13] The trend in model 3 shows a different pattern compared to the other models. This fact may be an indication of higher complexity in the links between unemployment and output, as mentioned in the text. (See also footnote 17 for additional problems regarding model 3.)

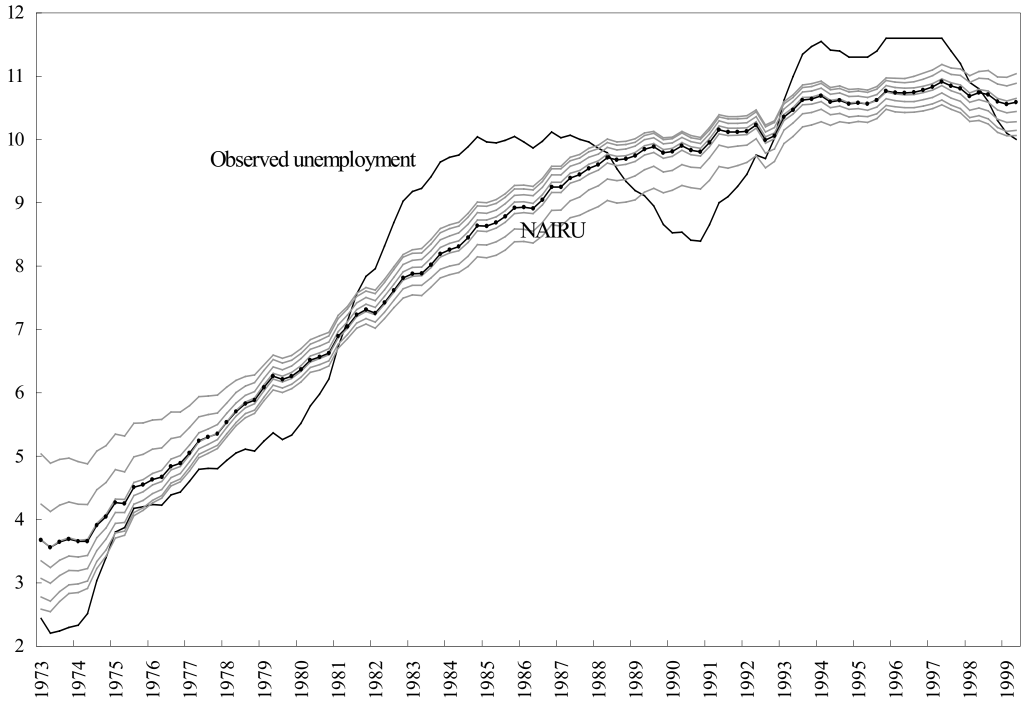
bands around the estimates. This is the option taken in this paper.[14] As already explained, the estimation strategy followed here precludes a direct impact of *a priori* assumptions on uncertainty, due to the non-informative prior distribution adopted. On the other hand, it is important to take full account of parameter uncertainty and sampling uncertainty.

The procedure finally chosen was to simulate artificial data sets according to the initially estimated parameters, and run the full estimation and state derivation exercise with each of the samples. After a number of tests, 500 simulations were seen as sufficient. Draws were derived using standard bootstrap techniques. Finally, uncertainty was measured by the relevant percentiles of the final generated estimates.

Bootstrapped confidence bands around estimated parameters for the last period for which an estimate could be derived are shown in the table in the previous section; uncertainty around the estimated NAIRU is shown in figures 3 and 4 below.

Figure 3 - NAIRU estimates and bootstrapped percentiles for the baseline model

(percentiles: 5%, 10%, 25%, 50%, 75%, 90%, 95% - 500 bootstraps)



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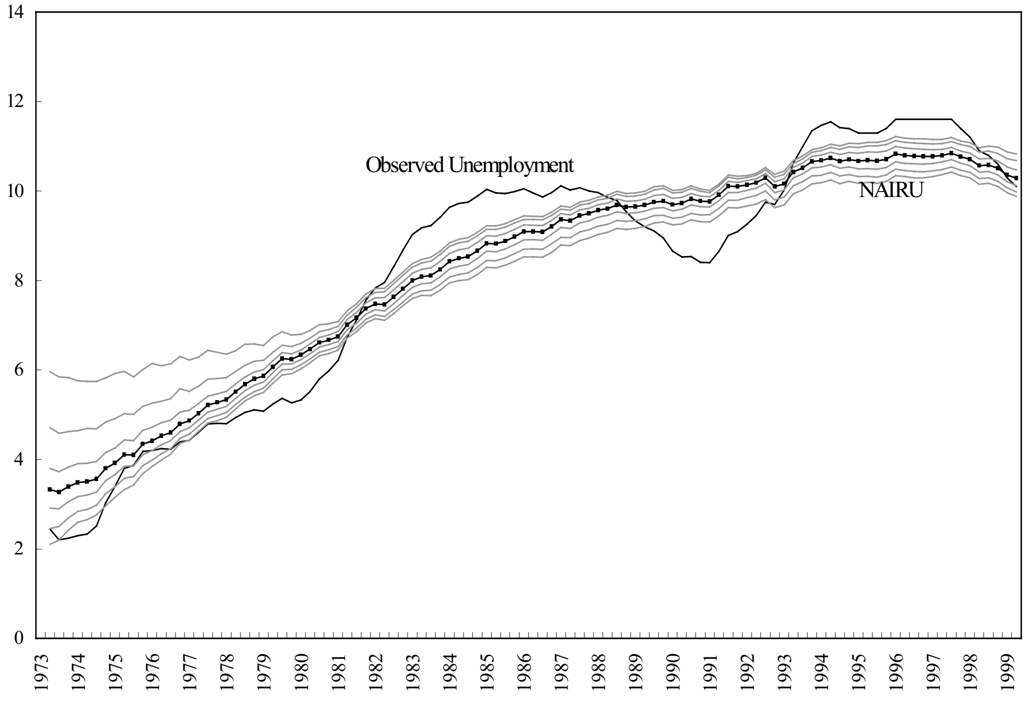
[14] This choice is not devoid of problems. In order to simplify the exercise, it was decided to avoid using complex Bayesian sampling techniques, see Bauwens et al. (2000) for a thorough survey of them, nor boostrap bias-correction was attempted.

The graph presented in figure 3 depicts chosen sample percentiles of the derived NAIRU for the baseline model, with percentiles covering the 5% to 95% range.[15] The four alternative specifications of the baseline model discussed in the previous sections provided similar results. Figure 4 shows the bootstrapped percentiles obtained by pooling together the results of the simulation exercises for the five models.[16]

Figure 4 – Bootstrapped percentiles: pooling of all models

(percentiles: 5%, 10%, 25%, 75%, 90%, 95%)



A few points are worth emphasising. First, all the models taken into consideration give a broadly similar picture in terms of the look of the estimated NAIRU.[17] Second, bands around the NAIRU are also similar, and of width which seems reasonable. In all cases, estimates of the NAIRU within the 5% lower and upper tails are comprised within around 1 percentage point. Third, with the exception of the initial years of the sample, uncertainty about the point at which observed unemployment and the NAIRU cross

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[15] Percentiles shown are for the 5%, 10%, 25%, 50%, 75%, 90% and 95% tails.

[16] In Figure 4, we define as NAIRU the median of the distribution obtained by pooling together the results of the five simulations.

[17] Only the model labelled as Model 3 gave some problems inherent into its specification. These problems were apparent in terms of convergence problems in the estimation and of higher uncertainty at the beginning of the sample, compared to the rest of the models.

covers slightly more than one year. Finally, crossings are rare and well separated in time, as could be expected from a series with the high persistence shown by European unemployment. As a consequence, periods in which unemployment was below or above the estimated NAIRU are clearly outlined.[18]

Another feature worth pointing out is the gap between the sample estimates for the NAIRU and the median derived from the simulations (figure 3). This was something to be expected, due to the potentially complex nature of the posterior distribution simulated. It can be checked that the estimated NAIRU crosses the median a number of times, although it is never far from it. It is very difficult to assess the source of this asymmetry, but a closer look at the estimated hyper-parameters of the systems (see previous section) clearly shows that estimated variances may be adding more to this gap than other parameters, by displaying some signs of asymmetry.[19]

5. CONCLUSIONS

This study has focused on estimating measures of the NAIRU for the euro area treated as a single entity. It has done so within an empirical framework widely used in the literature in recent years, but following an approach aimed at testing the robustness of the framework itself. The analysis has been based on systems of equations comprising a Phillips-curve, in which unobserved cyclical factors impinge on inflation, an Okun-type relationship, linking cyclical output and cyclical unemployment, and specific laws of motions for the unobservable variables, namely the NAIRU and potential output. Such systems have been estimated using Kalman-filter techniques.

The aim of this approach has been double: on the one hand, we have exploited recent advances in the mentioned techniques; on the other hand, we have tested the robustness of the approach by looking at alternative versions of the systems.

The results we have obtained point to area-wide NAIRU estimates that are robust to changes in the underlying models, as long as the models belong to a specific class described below. This robustness is shown to hold both in terms of the mean—i.e., the shape of the resulting NAIRU—and the variance of the process, the latter being derived through bootstrap exercises using the models alone or pooled together.

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[18] Compare these graphs with, for instance, those contained in Staiger et al. (1996) for the US.

[19] Although it is in theory possible to get rid of the biases in bootstrap exercises, very often this proves to be difficult to implement. In the results presented no bias correction was attempted.

A number of problems that emerged in the initial stages of the analysis have been solved thanks to two structural assumptions, which hold across all the different specifications tested. These are the presence of a stochastic trend as a maintained feature of the modelled NAIRU and the joint determination, in the systems explored, of potential output and the NAIRU.

The first feature has traditionally been seen as problematic, as it could lead to a trended measure of the NAIRU. Instead, in our systems the stochastic trend plays a somewhat different and much more restricted role, as all models consistently show a smooth drop from positive numbers, i.e. a steadfastly increasing NAIRU, to small negative numbers. In our view, this behaviour may be linked to a shift in observed unemployment over the sample period considered, i.e. to the fact that unemployment went strongly up during the seventies and has since stayed at high levels, obviously with short-term oscillations. This shift is widely held to be determined by structural, supply-side factors in most of the literature.[20] Results, if granted, would point to an improving picture of the structural links between inflation and unemployment, very likely the result of labour market reform started in the recent years. Estimates without this implicit stochastic trend were clearly less plausible.

The other important feature has been the presence of potential output as a mechanism embedded in the system, together with the NAIRU. Both cyclical output and cyclical unemployment were needed to shrink the uncertainty around estimates of the NAIRU to acceptable levels, especially in the early years of the sample period. Uncertainty in the estimated models is shown to be comparable with evidence for the US, despite the widely different behaviour of observed unemployment in Europe, and is found to be broadly in the range of 1 percentage point. As observed for the US, confidence bands are wide enough to blur the analysis of the current level of the NAIRU. For instance, confidence bands are such that a full year may elapse before a crossing between the NAIRU and observed unemployment may be taken as highly likely, at least in the business cycles observed in the sample. Nevertheless, these crossings are in the sample used less frequent, and hence longer lasting, than corresponding crossings in the US. Thus, uncertainty may be less of an issue in European data.

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[20] Obviously, this claim cannot be sustained by the analysis herein, as no structural factors are explicitly taken into account. All these factors should be considered to be lumped together in the residual of the NAIRU transition equation.

The approach has provided an interesting insight into the past behaviour of unemployment and inflation in Europe. It has also pointed to some evidence that labour market reforms might be starting to have a positive effect at the macroeonomic level. Procedures used in the paper could be used to monitor this evidence over the next future, in order to confirm these developments.

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APPENDIX A - STATE-SPACE MODEL

The Kalman filter estimation is composed of two stages, a filtering procedure and a smoothing procedure. The first is a recursive process that constructs the estimates for period t building on the information on the observed variables available at t-1, minimising the forecast error by maximum likelihood. The second procedure smoothes the obtained estimate on the basis of the information available over the whole sample period.

In all cases and for all state variables, a loose, non-informative prior was chosen. Alternatives ranged from deriving explicit asymptotic priors for the stationary states, to explicitly estimating prior means (setting prior variances to zero). The chosen option has the advantage of being easily dealt with, while at the same time its impact on current values for the states is probably negligible. The numerical algorithm adopted for the maximum likelihood system estimation was the SIMPLEX procedure available in the econometric package RATS.[21]

In general, for Kalman filter estimation, a model is expressed in its state-space form. The latter is composed of two parts: the measurement and the transition system. In this appendix we provide an example of such systems, referred to the baseline model presented in section 2.

Measurement system - the equations defining it are the following:

$$\begin{aligned}
y_t &= y_t^* + (y - y^*)_t \\
u_t &= u_t^* + (u - u^*)_t \\
\Delta\pi_t &= \alpha + a(L)\Delta\pi_{t-1} + \rho(L)(u_{t-1} - u_{t-1}^*) + b(L)z_t + \varepsilon_t^\pi
\end{aligned}$$

which can be summarised in matrix form as:

$$X(t) = M * U(t) + W * G(t) + E(X)$$

where

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[21] The RATS code developed to perform the maximum likelihood system estimation largely draws on programs which were kindly made available to the authors by Per Jansson, from the Sveriges Riksbank.

$$X(t)=\begin{bmatrix} y_t \\ u_t \\ \Delta\pi_t \end{bmatrix} \qquad M=\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}$$

$$W=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & b_1 & b_2 & b_3 & b_4 & \alpha \end{bmatrix}$$

$$U(t)=\begin{bmatrix} y_t^* \\ \beta_t \\ (y-y^*)_t \\ (y-y^*)_{t-1} \\ u_t^* \\ \xi_t \\ (u-u^*)_t \\ (u-u^*)_{t-1} \end{bmatrix} \qquad G(t)=\begin{bmatrix} \Delta\pi_{t-1} \\ \Delta\pi_{t-2} \\ \Delta\pi_{t-1} \\ \Delta\pi_t^M \\ \Delta\pi_{t-1}^M \\ \Delta\pi_{t-2}^M \\ \Delta\pi_{t-3}^M \\ 0 \end{bmatrix} \qquad E(X)=\begin{bmatrix} 0 \\ 0 \\ \varepsilon^{\pi} \end{bmatrix}$$

where the variance-covariance matrix of $E(X)$ is: $\Sigma_{E(X)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\sigma^{\pi})^2 \end{bmatrix}$

Transition system – the equations defining it are the following:

$$\begin{aligned}
y_t^* &= y_{t-1}^* + \beta_{t-1} + \varepsilon_t^{y*} \\
\beta_t &= \beta_{t-1} + \varepsilon_t^{\beta} \\
(y-y^*)_t &= \phi(L)(u-u^*)_{t-1} + \varepsilon_t^{yc} \\
u_t^* &= u_{t-1}^* + \xi_{t-1} + \varepsilon_t^{u*} \\
\xi_t &= \xi_{t-1} + \varepsilon_t^{\xi} \\
(u \ \ -u^*)_t &= \delta(L)(u-u^*)_{t-1} + \varepsilon_t^{uc}
\end{aligned}$$

which can be summarised in matrix form as:

$$U(t) = A * U(t-1) + E(U)$$

where

$$A=\begin{bmatrix}1&1&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&0&0&0&0&\phi_1&0\\0&0&1&0&0&0&0&0\\0&0&0&0&1&1&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&\delta_1&\delta_2\\0&0&0&0&0&0&1&0\end{bmatrix}\qquad E(U)=\begin{bmatrix}\varepsilon^{y*}\\\varepsilon^{\beta}\\\varepsilon^{yc}\\0\\\varepsilon^{u*}\\\varepsilon^{\xi}\\\varepsilon^{uc}\\0\end{bmatrix}$$

where the variance-covariance matrix of E(U) is:

$$\Sigma_{\mathrm{E(U)}}=\begin{bmatrix}(\sigma^{y*})^2&0&0&0&0&0&0&0\\0&(\sigma^{\beta})^2&0&0&0&0&0&0\\0&0&(\sigma^{yc})^2&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&(\sigma^{u*})^2&0&0&0\\0&0&0&0&0&(\sigma^{\xi})^2&0&0\\0&0&0&0&0&0&(\sigma^{uc})^2&0\\0&0&0&0&0&0&0&0\end{bmatrix}$$

APPENDIX B - THE EXTENDED TIME-VARYING-PARAMETER MODEL

It is well known that standard time-varying models in which the unobserved component is modelled as a simple random walk cannot be estimated unless the variance of the noise is imposed. One way to circumvent this problem is by increasing the amount of structure in the model in order to pin down this parameter, a strategy normally implemented by assuming specific dynamic forms for the cyclical (i.e., stationary) part of the state variable. Usually, an AR(1) or AR(2) process is assumed for the purpose. In our case, starting from a simple time-varying NAIRU in the form of a local linear trend model, this would lead to a system like (B.1).

$$\Delta\pi_t = \alpha + a(L)\Delta\pi_{t-1} + \rho(L)(u_{t-1} - u^*_{t-1}) + b(L)z_t + \varepsilon^{\pi}_t \tag{B.1a}$$

$$u^*_t = u^*_{t-1} + \xi_{t-1} + \varepsilon^{u*}_t \tag{B.1b}$$

$$\xi_t = \xi_{t-1} + \varepsilon^{\xi}_t \tag{B.1c}$$

$$(u - u^*)_t = \delta(L)(u - u^*)_{t-1} + \varepsilon^{uc}_t \tag{B.1d}$$

Expression (B.1d) in the system is an addition to an otherwise standard Phillips curve cum stochastic trend system, expressing one further state variable. The interplay between the three state variances of the problem (together with the rest of the parameters) is such that a better streamlining of the NAIRU variance is possible. This strategy was followed in this study, but was found unsuitable precisely because of the relatively poor estimation accuracy that was obtained. Results were interesting on their own, but not to the point of deserving being reported in the main text. Instead, this appendix reports results and justifies the more elaborate structures adopted finally.

In its state-space form, the model can be reformulated as follows.

- The measurement equation is: $X(t) = M * U(t) + W * G(t) + E(X)$

where $X(t) = \begin{bmatrix} u_t \\ \Delta\pi_t \end{bmatrix}, M = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \rho \end{bmatrix}, W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & b_1 & b_2 & b_3 & b_4 & \alpha \end{bmatrix}$

$$U(t)=\begin{bmatrix} u_t^* \\ \xi_t \\ (u-u^*)_t \\ (u-u^*)_{t-1} \end{bmatrix}, G(t)=\begin{bmatrix}\Delta\pi_{t-1} & \Delta\pi_{t-2} & \Delta\pi_{t-3} & \Delta\pi_t^M & \Delta\pi_{t-1}^M & \Delta\pi_{t-2}^M & \Delta\pi_{t-3}^M & 0\end{bmatrix}',$$

and the variance-covariance matrix of $E(X)$ is: $\Sigma_{E(X)} = \begin{bmatrix} 0 & 0 \\ 0 & (\sigma^{\pi})^2 \end{bmatrix}$

- The transition equation is: $U(t) = A * U(t-1) + E(U)$

where $A=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \delta_1 & \delta_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $E(U)=\begin{bmatrix} \varepsilon^{u*} \\ \varepsilon^{\xi} \\ \varepsilon^{uc} \\ 0 \end{bmatrix}$

and the variance-covariance matrix of E(U) is: $\Sigma_{E(U)} = \begin{bmatrix} (\sigma^{u*})^2 & 0 & 0 & 0 \\ 0 & (\sigma^{\xi})^2 & 0 & 0 \\ 0 & 0 & (\sigma^{uc})^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The above model was estimated by maximum likelihood using the SIMPLEX optimisation algorithm in RATS, as done for all models in the main text. The results obtained for the relevant parameters of the measurement and transition equations and for the unobserved NAIRU are presented in table B.1.

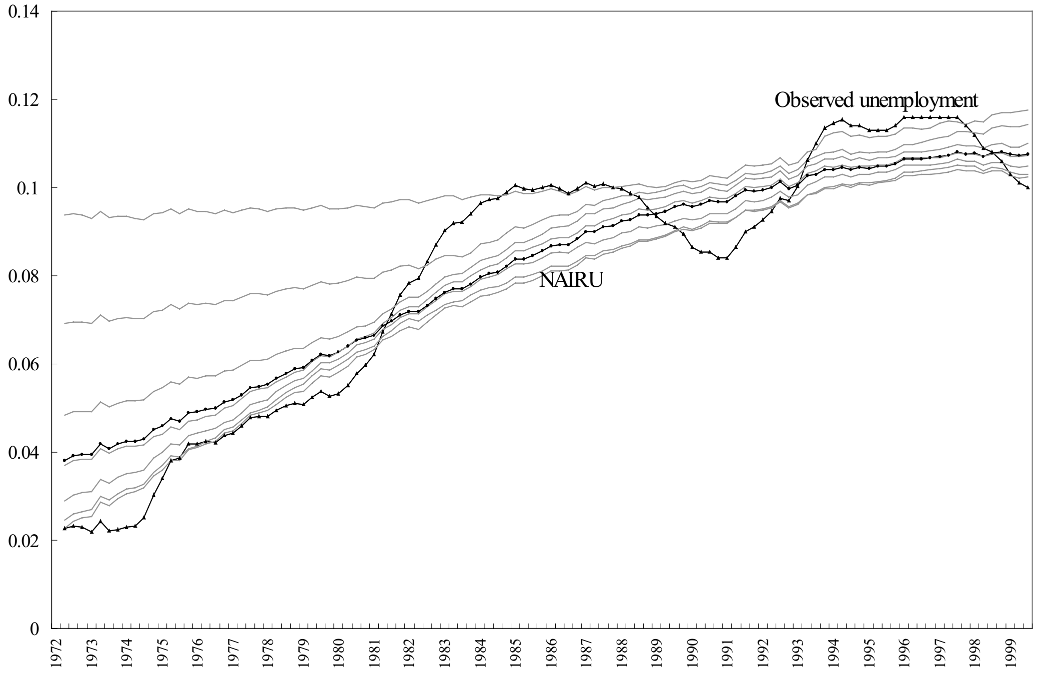
Results are apparently reasonable and well in line with results reported elsewhere in the study. But bootstrap exercises performed with the system led to the conclusion that the degree of uncertainty around parameters was too high to deem the system as really suitable. This point is made evident on figure B.1, in which bands around the NAIRU estimate, following the same steps as in the main text, are drawn. NAIRU uncertainty in the graph is visibly higher than similar exercises performed for the alternative models, particularly for the initial periods. In view of that, this particular model was not included in the main text.

Table B.1: System estimates for the extended TVP model

Parameters	
Phillips curve	
ρ	*-0.051*
a_1	*-0.542*
a_2	*-0.207*
a_3	*-0.155*
b_1	*0.079*
b_2	*0.067*
b_3	*0.036*
b_4	*0.074*
Autocorrelation$^{(1)}$	*8.20 (0.61)*
Normality$^{(2)}$	*1.22 (0.54)*
Cyclical fluctuations	
δ_1	*1.796*
δ_2	*-0.813*
Standard deviations	
σ^{π}	*0.0020*
σ^{u^*}	*0.0009*
σ^{ξ}	*0.0001*
NAIRU in 1999Q3	*10.77* *(10.24-11.76)*$^{(3)}$

Notes: (1) Ljung-Box Q test measuring general AR(10) residual autocorrelation (p-values in parentheses). (2) Doornik-Hansen normality test (p-values in parentheses). (3) The interval represents the 95% confidence band obtained by bootstrapping techniques.

Figure B.1 - NAIRU estimates and bootstrapped percentiles for the extended TVP model



APPENDIX C - POTENTIAL OUTPUT ESTIMATES

The paper has not focused on the estimation of a measure of potential output for the euro area. The introduction of potential output and of the output gap in the models considered in the empirical analysis has been aimed, as we stated in the main text, at better pinning down the relevant parameters of the systems to be estimated, by increasing the structural information contained in the systems themselves. However, for sake of completeness, in figure C.1 we report the estimate of potential output which has been used in the baseline model to derive the aggregate NAIRU, compared with observed output and with a smoothed version of it based on the Hodrick-Prescott filter.

Figure C.1 – Potential output

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