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ECB CONFERENCE ON MONETARY POLICY AND IMPERFECT KNOWLEDGE

PARAMETER MISSPECIFICATION AND ROBUST MONETARY POLICY RULES

by Carl E. Walsh

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Abstract

In this paper, I evaluate the performance deterioration that occurs when the central bank employs an optimal targeting rule that is based on incorrect parameter values. I focus on two parameters – the degree of inflation inertia and the degree of price stickiness. I explicitly account for the effects of the structural parameters on the objective function used to evaluate outcomes, as well as on the model's behavioral equations. The costs of using simple rules relative to the costs of parameter misspecification are also assessed.

Keywords: Monetary Policy, Robustness, Misspecification
JEL Classification: E52, E58

Non–technical summary

In this paper, I evaluate the effects of parameter misspecification on the class of robustly optimal targeting rules implied by the first order conditions of the central bank's decision problem. In contrast to most analyzes of parameter uncertainty, I incorporate the consequences of parameter misspecification both on the policy maker's perceptions of the structural equations governing the determination of the endogenous variables and on the loss function the central bank employs in deriving its optimal policy. This contrasts with previous research where macroeconomic outcomes are evaluated under the assumption that the policy maker has a fixed (and known) objective function. As recent theoretical work has shown, however, the appropriate policy objective function itself will depend on the structural characteristics of the economy. Thus, misspecification of the model's structural parameters will also imply the objective function is misspecified.

I focus on misspecification of two key parameters - the degree of structural inflation inertia and the degree of nominal price stickiness. There is great uncertainty about the true values of each of these parameters, and each is likely to have a significant impact on policy design. Previous research has found that it may pay central banks to err in the direction of overestimating inflation persistence. However these results are based on using a fixed objective function to evaluate outcomes.

I find that the optimal targeting rule is robust to misspecification of inflation inertia. In part, this result arises because previous work has ignored the impact of inflation inertia on the social loss function. The rule is not as robust to misspecifying the degree of nominal price stickiness. However, I find that an assessment of the costs of misspecifying the degree of price stickiness depends significantly on whether one also accounts for the effects on the objective function that measures social welfare.

After assessing the robustness of robustly optimal targeting rules, I examine the cost of using a simple rule in the face of parameter misspecification. I decompose this cost into two components: the effects of parameter misspecification, conditional on implementing an optimal policy, and the effects of employing a simple rule in place of the fully optimal policy.

I focus on optimal difference rules as an example of a simple policy rule that has previously been shown to perform well in a variety of models. For uncertainty about the degree of inflation inertia, the simple rule was more robust than the optimal targeting rule. However, for most combinations of actual and perceived structural inflation inertia, the loss from using a simple rule rather than the optimal targeting rule exceeded the costs arising from parameter misspecification. When the degree of nominal rigidity is potentially misspecified, the simple rule's performance deteriorated significantly if the policy maker thinks prices are relatively flexible when in fact they are quite sticky. For most combinations of actual and perceived price stickiness, however, the loss from using a simple rule rather than the optimal targeting rule exceeded the costs arising from parameter misspecification.

1 Introduction

One objective of the large literature that has investigated uncertainty and monetary policy has been to find robust rules, rules that perform well even when the policy maker has imperfect knowledge of the true structural equations that characterize the economy. One approach in this literature focuses on model uncertainty and the performance of policy rules across different models. McCallum (1988, 1999) has long argued for evaluating policy proposals in a variety of economic models as a means of assessing their robustness, and Levin and Williams (2003a) and Levin, Wieland, and Williams (2003) explore the performance of simple rules calibrated to be optimal in one model when the true economy is described by a different model. However, most research has focused on uncertainty with respect to a given reference model. For example, Brainard (1967) examined how the optimal instrument rule of a Bayesian decision maker is altered when faced with parameter uncertainty.[1] Using a new Keynesian model, Giannoni and Woodford (2003a, 2003b) have analyzed instrument rules that are robust to misspecification of the disturbance processes of a known model, while Hansen and Sargent (2003) derive policies that are designed to be robust in the sense of minimizing the worst case scenario when the policy maker believes the true model is in a neighborhood of a given reference model.

In this paper, I evaluate the effects of parameter misspecification on the class of robustly optimal targeting rules proposed by Svensson and Woodford (2003) and Giannoni and Woodford (2003a, 2003b). In contrast to most analyzes of parameter uncertainty, I incorporate the consequences of parameter misspecification both on the policy maker's perceptions of the structural equations governing the determination of the endogenous variables and on the loss function the central bank employs in deriving its optimal policy. This contrasts with previous research where macroeconomic outcomes are evaluated under the assumption that the policy maker has a fixed (and

[1] Brainard (1967) showed how multiplicative uncertainty could lead to policy attenuation. Craine (1979) and Södeström (2002) demonstrated that Brainard's result does not hold generally, and that uncertainty can lead to more aggressive policy responses in some cases.

known) objective function. As recent theoretical work has shown, however, the appropriate policy objective function itself will depend on the structural characteristics of the economy (Woodford 2003). Thus, misspecification of the model's structural parameters will also imply the objective function is misspecified.

To date, Kimura and Kurozumi (2003), Kurozumi (2003), and Levin and Williams (2003b) have incorporated the effects of structural parameters on the loss function into a Bayesian analysis of parameter uncertainty. These authors focus on whether parameter uncertainty leads to more cautious or more aggressive policy responses to shocks. Kimura and Kurozumi (2003) and Kurozumi (2003) find that, in contrast to the traditional attenuation of policy found by Brainard (1967), responses tend to be more aggressive. Levin and Williams show that multiplicative uncertainty about the elasticity of inflation with respect to output may not lead to the traditional policy attenuation when the effects on the objective function are taken into account. In contrast to these papers, I follow the approach of Angeloni, Coenen, and Smets (2003) in focusing on what Coenen (2003) calls "uncertainty about the rule-generating model." That is, I investigate the consequences of basing the optimal targeting rule on misspecified parameters.

I focus on misspecification of two key parameters - the degree of structural inflation inertia and the degree of nominal price stickiness. There is great uncertainty about the true values of each of these parameters, and each is likely to have a significant impact on policy design. Coenen (2003) and Angeloni, Coenen, and Smets (2003) have found that it may pay central banks to error in the direction of overestimating inflation persistence, a result also found by Walsh (2003c). However these results are based on using a fixed objective function to evaluate outcomes.

After assessing the robustness of robustly optimal targeting rules, I examine the cost of using a simple rule in the face of parameter misspecification. I decompose this cost into two components: the effects of parameter misspecification, conditional on implementing an optimal policy, and the effects of employing a simple rule in place of the fully optimal policy.

The basic model, developed in the next section, is a standard new Keynesian model with inflation inertia (Woodford 2003). In section 3, the robustly optimal targeting rule of Giannoni and Woodford (2003a, 2003b) is derived. The effects of parameter misspecification are investigated in section 4. In section 5, the costs of employing a simple rule are compared to the costs of parameter misspecification. Conclusions are summarized in section 6.

2 Basic model

The foundations of the benchmark new Keynesian model have been discussed extensively; I draw heavily on the formulation of Woodford (2003) to which the reader is referred.[2] The first equation of the model relates the output gap x_t (output relative to the flexible-price equilibrium level of output) to its expected future value and the real interest rate gap, the difference between the actual real interest rate and the natural real rate r_t^n:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t \pi_{t+1} - r_t^n\right), \tag{1}$$

where i_t is the nominal rate of interest and π_{t+1} is the inflation rate from t to $t+1$. For simplicity, assume r^n is exogenous and evolves according to

$$r_t^n = \rho_r r_{t-1}^n + v_t,\ 0 \leq \rho_r < 1. \tag{2}$$

The innovation v_t is white noise.

The second structural equation is an inflation adjustment equation. As is well known, purely forward-looking models of inflation generally fail to capture the empirical persistence actual inflation seems to display. Various modifications that allow for greater inflation persistence while preserving the tractability of the Calvo specification have been explored in the literature.[3] I follow Woodford (2003) in assuming that with probability $1-\alpha$ firms optimally adjust their price and with probability α they simply index their

[2] See Walsh (2003a, Chapters 5 and 11) for further discussion of this model and additional references to the literature.

[3] For example, see Galí and Gertler (1999), Christiano, Eichenbaum, and Evans (forthcoming), and Eichenbaum and Fisher (2004).

price by a fraction γ, $0 \leq \gamma \leq 1$, of the most recent rate of inflation. This leads to an inflation equation of the form

$$\pi_t - \gamma\pi_{t-1} = \beta \left(E_t\pi_{t+1} - \gamma\pi_t \right) + \kappa x_t + e_t, \tag{3}$$

where e is a cost shock that captures any factors that alter the relationship between real marginal costs and the output gap.[4] The cost shock is taken to be exogenous, given by

$$e_t = \rho_e e_{t-1} + \varepsilon_t, \; 0 \leq \rho_e < 1. \tag{4}$$

The innovation ε is white noise. In (3), the parameter β is the discount rate, while κ is a function of the parameters governing the frequency of price adjustment, the elasticity of marginal cost with respect to output, and the demand elasticity faced by individual producers. Specifically, if ω is the elasticity of the representative agent's utility with respect to output and ψ is the price elasticity of demand facing the individual firm, then[5]

$$\kappa = \left[\frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right] \left(\frac{\varepsilon_{mc}}{1+\omega\psi} \right)$$

where ε_{mc} is the elasticity of real marginal cost with respect to output.

The system consisting of (1) - (4) is closed by a specification of monetary policy. The central bank's objective is to minimize a loss function that depends on the variation of inflation, the output gap, and the nominal interest rate:

$$L_t = \left(\frac{1}{2} \right) E_t \sum_{i=0}^{\infty} \beta^i \left[z_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i (i_{t+i} - i^*)^2 \right], \tag{5}$$

where $z_t \equiv \pi_t - \gamma\pi_{t-1}$. Woodford (2003) discusses the conditions under which (5) can be viewed as proportional to a second order approximation to the utility of the representative agent. He also shows that the

[4] As Woodford (2003, Ch. 6) shows, if real money balances and consumption are nonseparable in utility, a term involving the nominal rate of interest also appears in the inflation adjustment equation. I follow Giannoni and Woodford (2003b) in ignoring this effect.

[5] See Woodford (2003).

weights λ_x and λ_i are functions of the underlying structural parameters that describe preferences and price adjustment. Specifically, $\lambda_x = \kappa/\psi$ and $\lambda_i = (\bar{m}/\bar{Y})(\eta_i \lambda_x / \varepsilon_{mc})$, where $\bar{m}/\bar{Y}$ is the steady-state ratio of real money balances to output, η_i is the interest rate semi-elasticity of money demand, and $\varepsilon_{mc} = \sigma + \omega - \chi\eta_y$, where η_y is the income elasticity of money demand and χ is the elasticity of marginal utility with respect to real money holdings.

3 Robustly optimal targeting rules

Svensson and Woodford (2003) and Giannoni and Woodford (2003a, 2003b) have analyzed a class of policy rules that Giannoni and Woodford describe as *robustly optimal*. These rules are optimal in that the rule supports the equilibrium consistent with an optimal commitment policy when evaluated from the timeless perspective (Woodford 2003), and they are robust in that the coefficients in the policy rule are independent of the parameters that characterize the behavior of the exogenous, stochastic disturbances. Thus, the policy maker implementing such a rule does not need to know whether disturbances are highly persistent or transitory or whether demand shocks are more volatile than cost shocks.

The optimal commitment policy in the model consisting of (1) - (5) has been well studied (Giannoni and Woodford 2003b, Svensson and Woodford 2003, Woodford 2003). Under commitment, the Lagrangian for the policy maker's decision problem is

$$\begin{aligned} L_t \quad = \quad & E_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{1}{2}\right) \left[z_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i (i_{t+i} - i^*)^2 \right] \right. \\ & s_{1t+1+i} \left(z_{t+i} - \beta z_{t+1+i} - \kappa x_{t+i} - e_{t+i} \right) \\ & + s_{2t+1+i} \left[x_{t+i} - x_{t+1+i} + \left(\frac{1}{\sigma}\right) \left(i_{t+i} - \pi_{t+1+i} - r_{t+i}^n \right) \right] \\ & \left. + s_{3t+1+i} \left(z_{t+i} - \pi_{t+i} + \gamma \pi_{t-1+i} \right) \right\}, \end{aligned} \tag{6}$$

where s_1, s_2, and s_3 denote Lagrangian multipliers. The necessary first order conditions include (1) - (4) and the following four equations:

$$z_t + s_{1t+1} - s_{1t} + s_{3t+1} = 0, \tag{7}$$

$$\lambda_x x_t - \kappa s_{1t+1} + s_{2t+1} - \left(\frac{1}{\beta}\right) s_{2t} = 0, \tag{8}$$

$$-\left(\frac{1}{\beta\sigma}\right) s_{2t} - s_{3t+1} + \beta\gamma E_t s_{3t+2} = 0, \tag{9}$$

$$\lambda_i (i_t - i^*) + \left(\frac{1}{\sigma}\right) s_{2t+1} = 0. \tag{10}$$

Under the fully optimal commitment policy, the multipliers s_{1t} and s_{2t} are set to zero, as no previous commitments are binding in the first period, and both s_{1t+1} and s_{2t+1} are functions of the time t predetermined values. Woodford has argued for adopting a "timeless perspective" in which s_{1t} and s_{2t}, rather than being set equal to zero, satisfy the same first order conditions as the current and future multipliers do. Specifically, assume the optimal commitment policy from a timeless perspective has been in place since at least $t-2$. Giannoni and Woodford (2003b) show that this assumption allows one to eliminate the Lagrangian multipliers and obtain a targeting rule of the form

$$E_t A(L) i_{t+1} = \left(\frac{\kappa}{\beta\sigma}\right) i^* - \left(\frac{\kappa}{\sigma\lambda_i}\right) \left[q_t - \beta\gamma E_t q_{t+1}\right], \tag{11}$$

where $A(L)$ *is* a polynomial in the lag operator L given by

$$A(L) = \beta\gamma - (1+\gamma+\beta\gamma) L + \left[1+\gamma+\frac{1}{\beta}\left(1+\frac{\kappa}{\sigma}\right)\right] L^2 - \left(\frac{1}{\beta}\right) L^3,$$

and $q_t \equiv z_t + (\lambda_x/\kappa)(x_t - x_{t-1})$.[6] Equation (11) is the Giannoni-Woodford robustly optimal rule. As they emphasize, the coefficients in (11) do not depend on either the serial correlation coefficients ρ_r and ρ_e or on the variances of the innovations to the disturbances. It is, therefore, robust with respect to misspecification of these aspects of the model. The equilibrium

[6] See the appendix for details.

under the optimal (timeless perspective) commitment policy is given by the solution to equations (1) - (4) and (11).

As long as $\lambda_i \neq 0$, (11) can be expressed as an instrument rule for the nominal rate of interest, although it is not an explicit recipe for setting i_t since it depends both on other contemporaneous endogenous variables and on expectations of the future value of i itself. In fact, (11) is a form of the first order condition for the central bank's optimal policy problem, and Svensson (2004) has argued for describing such conditions as targeting rules rather than instrument rules. This more general terminology recognizes that even when $\lambda_i = 0$, so that the instrument i_t does not appear directly in the first order condition, one can still derive a robustly optimal rule.[7] Woodford (2003) also describes a relationship such as (11) as a target criterion.

Woodford (2003) stresses the robustness of the targeting rule given by (11) to misspecification of the disturbance processes, but, interestingly, it is also the targeting rule that implements the optimal robust policy in the robust control sense of Hansen and Sargent (2004). In a series of papers, Hansen and Sargent, together with coauthors, have explored robust control approaches to the decision problem of agents who face model uncertainty (Hansen and Sargent 2003, 2004). In the approach they develop, the central bank views its model as an approximation to the true model of the economy, knowing only that the true model is in a neighborhood around its approximating model.[8] Robust policies, in the sense of Hansen and Sargent, are min-max policies, designed to perform well in worst-case scenarios.[9] Despite this contrast with the standard rational expectations approach to optimal

[7] In the present case, if $\lambda_i = 0$, the central bank's first order conditions imply that the following condition must hold:

$$\pi_t - \gamma\pi_{t-1} = -\left(\frac{\lambda_x}{\kappa}\right)(x_t - x_{t-1}).$$

[8] Alternative approaches to robust control have been explored by Stock (1999), Giannoni (2002), Onatski and Stock (2002), and Onatski and Williams (2003).

[9] Hansen and Sargent (2004, Chapter 15) apply their robust control methodology to Woodford's forward-looking new Keynesian model, and Giordani and Söderlind (2003) report some simulations of a basic new Keynesian model under robust min-max policies. Some policy implications of this approach are discussed in Walsh (2003c).

policy, the targeting rule (11) also minimizes loss in the Hansen-Sargent worst case scenario when the public's and the central bank's expectations coincide. This equivalence of policy rules is demonstrated for a basic new Keynesian model without structural inflation inertia and with $\lambda_i = 0$ in Walsh (forthcoming); this equivalence extends to the present model.

While (11) characterizes optimal policy in either the standard (timeless perspective) optimal commitment case or in the min-max robust control analysis of Hansen and Sargent, the equilibrium behavior of inflation and output differs in the two cases. It does so because expectations are formed differently. The standard policy problem that Giannoni and Woodford analyze assumes expectations are formed rationally. In the Hansen and Sargent approach, expectations about future outcomes are based on worst-case scenarios.

The equivalence of targeting rules under alternative approaches to uncertainty suggests that the practice of characterizing the effects of model uncertainty in terms of whether it leads the policy maker to react more cautiously or more aggressively to inflation and output may fail to adequately capture the impacts of uncertainty. For example, implementing (11) achieves the optimal policy of a policy maker with a concern for robustness in the sense of Hansen and Sargent, but the coefficients in the rule are independent of the policy maker's preference for robustness. While two policy makers may have very different preferences for robustness, both could implement their optimal policies by ensuring that (11) holds. Hence, differing attitudes towards uncertainty may not always affect the way in which policy should be optimally adjusted to current and expected future values of inflation and the output gap.

4 Parameter robustness

While the robustly optimal rule is designed to protect against misspecification in the shock processes, the coefficients of the targeting rule depend on the underlying structural parameters of the model. Thus, implementing the rule requires knowledge of the structural parameters. In this section, I focus on outcomes when the policy maker implements an optimal rule but bases the rule on parameter values that are misspecified. Examining the sensitivity of macroeconomic outcomes to parameter misspecification provides one means of assessing the robustness of a rule. I focus on two parameters for which a wide range of values have been employed in the literature. These are the degree of structural inflation inertia, γ, and the degree of price stickiness, α.

By misspecifying a structural parameter, the policy maker is subject to two types of errors. First, the structural parameters affect the coefficients in the policy rule, so employing incorrect values for these parameters directly distorts the coefficients in the policy rule. This source of error has been the focus of previous work; for example Coenen (2003) and Angeloni, Coenen and Smets (2003) investigate the consequences of employing a rule that is optimized for an incorrect representation of inflation persistence.

However, there is a second source of potential error; the loss function on which policy is based may be incorrect, as it is itself a function of the structural parameters. The loss function may be misspecified because the wrong variables appear in it. For instance, if the degree of structural inflation inertia is misspecified, the wrong quasi-difference of inflation will appear in the loss function. Or it may be misspecified because the weights assigned to different objectives may be wrong; if the degree of nominal rigidity is misspecifed, the weights λ_x and λ_i will be incorrect.

Suppose the policy maker implements the rule that is optimal based on a perceived set of parameter values. Denote these values with a superscript P. One approach to investigating the impact of parameter misspecification is to solve the model consisting of the structural equations (1) and (4) and the policy rule given by

$$E_t A^P(L) i_{t+1} = \left(\frac{\kappa^P}{\beta\sigma}\right) i^* - \left(\frac{\kappa^P}{\sigma\lambda_i^P}\right)\left[q_t^P - \beta\gamma^P E_t q_{t+1}^P\right], \qquad (12)$$

where $A^P(L) = \beta\gamma^P - \left(1+\gamma^P+\beta\gamma^P\right)L + \left[1+\gamma^P+\frac{1}{\beta}\left(1+\frac{\kappa^P}{\sigma}\right)\right]L^2 - \left(\frac{1}{\beta}\right)L^3$ and $q_t^P \equiv \pi_t - \gamma^P\pi_{t-1} + \left(\lambda_x^P/\kappa^P\right)\left(x_t - x_{t-1}\right)$. Note that while I only consider misspecified values of α and γ, the value of α affects κ, λ_x, and λ_i, and the form of q_t depends on γ.

Implementing a misspecified policy rule when the optimal targeting rule involves expectations of future variables, as in (11), is not straightforward. As pointed out by Onatski and Williams (2004), simply combining a rule based on incorrect parameters with the true structural equations to solve for the rational expectations equilibrium would implicitly allow policy to be based on forecasts which, in turn, are based on a knowledge of the true structural coefficients.[10] An alternative approach is to recast the policy rule into a form that does not directly depend on expectations of future endogenous variables. In equilibrium, the nominal rate of interest will be a function of the model's predetermined variables, but, as is well known (Woodford 2003), employing a rule that only involves predetermined variables does not ensure determinacy.

To obtain an optimal instrument rule that is also consistent with a determinate solution, I adopt the following approach.[11] Let δ denote the vector of "true" structural parameter values. Let $Y_t = [e_t\ r_t^n\ \pi_{t-1}]'$ and let $\phi_t = [s_{1t}\ s_{2t}]'$, where s_1 and s_2 are the Lagrangian multipliers on the inflation adjustment and expectational IS equations respectively in the policy optimization problem (see 6). The appendix shows that when policy is based on the true δ, jointly solving (1) - (4) and (11) yields an equilibrium of the form

$$\begin{bmatrix} Y_{t+1} \\ \phi_{2t+1} \end{bmatrix} = \begin{bmatrix} M_{11}(\delta) & M_{12}(\delta) \\ M_{21}(\delta) & M_{22}(\delta) \end{bmatrix} \begin{bmatrix} Y_t \\ \phi_{2t} \end{bmatrix}, \tag{13}$$

where the notation indicates that the matrices that characterize the solution depend on the parameter vector δ. In addition, the equilibrium behavior of the Lagrangian multiplier s_3 in (6) is given by

$$s_{3t} = C_{31}(\delta)Y_t + C_{32}(\delta)\chi_{2t}. \tag{14}$$

[10] Alternatively, one might interpret the policy maker as employing empirical forecasting equations to form expectations and that the relevant equilibrium is one in which these forecasting equations coincide with rational expectations, even though the policy maker has incorrect estimates of the underlying structural parameters.

[11] See also Onatski and Williams (2004).

The first order conditions (7) - (10) can be used to obtain a policy rule of the form

$$\begin{aligned} i_t &= -\left(\frac{\kappa}{\sigma\beta}\right) i^* + \left(\frac{\kappa}{\sigma\lambda_i}\right) q_t + \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta\sigma}\right) i_{t-1} \\ &\quad -\left(\frac{1}{\beta}\right) i_{t-2} + \left(\frac{\beta\gamma\kappa}{\sigma\lambda_i}\right) E_t s_{3t+2}, \end{aligned} \tag{15}$$

which only involves expectations of the multiplier s_3. Using (13) and (14), one can write

$$E_t s_{3t+2} = C_{31}^*(\delta) Y_t + C_{32}^*(\delta)\phi_{2t},$$

where the matrices $C_{31}^*(\delta)$ and $C_{32}^*(\delta)$ are defined in the appendix.

Now let δ^P denote policy maker's perceived value for δ. The instrument rule based on the policy maker's perceived parameters is given by

$$\begin{aligned} i_t &= \left(\frac{\kappa^P}{\sigma\lambda_i^P}\right)\left[\pi_t - \gamma^P \pi_{t-1} + \frac{\lambda_x^P}{\kappa^P}(x_t - x_{t-1})\right] + \left(1 + \frac{1}{\beta} + \frac{\kappa^P}{\beta\sigma}\right) i_{t-1} \\ &\quad -\left(\frac{1}{\beta}\right) i_{t-2} - \left(\frac{\kappa^P}{\beta\sigma}\right) i^* + C_{31}^*(\delta^P) Y_t + C_{32}^*(\delta^P)\phi_{2t} \end{aligned} \tag{16}$$

while

$$\phi_{2t+1} = M_{21}(\delta^P) Y_t + M_{22}(\delta^P)\phi_{2t}. \tag{17}$$

Equilibrium under the misspecified parameters is obtained by solving for the rational expectations solution to (1) - (4), (16), and (17).

To investigate the impact of parameter misspecification, I solve a calibrated version of the model for $\alpha = \alpha^P$ and $\gamma = \gamma^P$ to obtain the matrices in (**??**) and (17). Then, holding the coefficients in these two equations fixed, I jointly solve (1) - (4), (16), and (17) for values of α (γ) in (1) and (3) ranging from 0.05 to 0.95 (0 to 1). I then calculate the value of the loss function using the weights λ_x and λ_i for the appropriate value of α (γ).

The baseline parameter values are taken from Giannoni and Woodford (2003b) and Woodford (2003) and are reported in Table 1.[12] While most

[12] All values are based on specifying both the structural equations and the loss function at quarter rates. Woodford typically specifies the structural equations at quarterly rates and the loss function in terms of inflation at annual rates. This would require multiplying

of these parameter values are fairly standard, the value for the coefficient of relative risk averse ($\sigma = 0.16$) is much lower than other researchers typically assume or the empirical evidence on the elasticity of output with respect to the real rate of interest suggests (e.g., Dennis 2003).[13] A more common value for σ would be in the range from 1 (log utility) to 5. Woodford's choice of 0.16 is based on estimates reported in Rotemberg and Woodford (1997).

Table 1: Parameter Values

Structural Parameters		Implied Values	
α	0.66	κ	0.024
β	0.99	λ_x	0.003
γ	0.50	λ_i	0.077
σ	0.16	ε_{mc}	0.63
ω	0.49		
ψ	7.88		
η_i	28	Innovations	
η_y	1	σ_v	0.0093
$\bar{v}$	7.25	σ_ε	0.0041
χ	0.02	ρ_r	0.35
		ρ_e	0

The unconditional expected value of the loss function given by (5) is denoted by $V(\delta, \delta^P, R)$, where R will index the form of the rule, either robustly optimal (ROR) or, in section 5, optimal simple rule (SR). The percentage deterioration in the loss function due to misspecification is measured by

$$\mu^*(\delta, \delta^P, R) = 100 \ln \left[\frac{V(\delta, \delta^P, R)}{V(\delta, \delta, ROR)} \right] \geq 0,$$

where $V(\delta, \delta, ROR)$ denotes the value of the social loss function as a function of the actual parameters when the optimal commitment policy rule is

λ_x by 16 (so that $\lambda_x = 0.048$, the value reported by Woodford).

[13] A value of $\sigma = 0.16$ implies a coefficient of 6.25 on the interest rate in the IS relationship. Expressing the interest rate at an annual rate would reduce this value to 1.56, although this is still well above most estimates.

implemented. The function $\mu^*(\delta, \delta^P, R)$ can serve to illustrate the regions of the parameter space over which misspecification is particularly costly. However, the unscaled function $\ln V(\delta, \delta^P, R)$ is also informative in that it can highlight whether it is beneficial to systematically err in the direction of over- or under-estimating a parameter.

4.1 Structural inflation inertia

The degree of structural inflation inertia has been the focus of a great deal of empirical research, and while this work finds persistent movements of inflation are a feature of the data, no consensus has been reached on the reason for it. Among the models commonly used for policy analysis are ones that assume only forward-looking inflation behavior (Woodford 2003) and ones that assume inflation is a purely backward-looking phenomenon (Rudebusch and Svensson 1999). The nature of the inflation process and the degree of endogenous structural inertia in the inflation process has been identified as one of the most critical parameters affecting the evaluation of alternative policies. For example, Rudebusch (2002) found that nominal income targeting does well when inflation is forward-looking but poorly when it is more backward-looking. Similarly, when current inflation is affected by both expected future inflation and lagged inflation, the performance of price-level targeting deteriorates significantly as the relative weight on lagged inflation rises (Walsh 2003b). Levin and Williams (2003a) demonstrate that policy rules that are optimal in a forward-looking model can lead to disastrous results if the true model is in fact backward-looking.

Unfortunately, given the significance it has for the evaluation of alternative policies, there is great uncertainty about the respective roles of forward and backward elements in the inflation process. For example, Rudebusch (2002) uses an output gap measure based on de-trended output and estimates the weight on lagged inflation to be over twice that on expected future inflation, while Galí and Gertler (1999), using a measure of real marginal cost rather than the output gap, find essentially the reverse.

In the present model, the degree of structural inflation inertia is measured by the parameter γ. To assess the costs of basing policy on an incorrect estimate of γ, figure 1 shows $\ln V(\gamma, \gamma^P, ROR)$, the (log) loss when the inflation equation and social welfare depend on γ but policy is characterized by the optimal targeting rule for a value γ^P. Along the diagonal, $\gamma = \gamma^P$, corresponding to the case of no specification error. In this case, loss is increasing in γ, reflecting the poorer trade-off faced by the policy maker as inflation becomes more backward-looking. The surface shown in the figure is fairly symmetric, indicating that when structural inflation inertia is misspecified, the costs of under-estimating γ are only slightly higher than those arising from over-estimating it. This differs from the findings reported in Walsh (2003c), where the costs of underestimating γ were found to be larger, and in Coenen (2003) and Angeloni, Coenen, and Smets (2003), who also find that it is better to over-estimate inflation inertia. However in those papers, a standard quadratic loss function was employed. In the present model, the loss depends on $\pi_t - \gamma\pi_{t-1}$ and so varies with γ. A policy maker concerned with minimizing the worst-case outcome would do best, however, by assuming inflation is relatively inertial; a value of $\gamma^P = 0.65$ minimizes $\max_\gamma \ln V(\gamma, \gamma^P, ROR)$.

Figure 2 plots $\mu^*(\gamma,\ \gamma^P, ROR)$, the percentage deterioration in loss. Along the diagonal, $\gamma = \gamma^P$, and $\mu^*(\gamma,\ \gamma^P, ROR) = 0$ by definition. The largest percentage deterioration occur when inflation is not persistent (γ is close to zero), but the central bank designs its policy under the assumption that inflation is predominately backward-looking (γ^P close to one). The maximum cost of misspecifying the degree of inflation inertia is just under 11%; thus, the costs of misspecifying the degree of inflation inertia are relatively small under the ROR rule.

4.2 The degree of price stickiness

The degree of price stickiness, α, affects both the structural parameter κ that appears in the inflation adjustment equation and, through κ, the weights λ_x and λ_i. In most previous work analyzing the robustness of policy to uncertainty about model parameters, changes in the structural parameters were

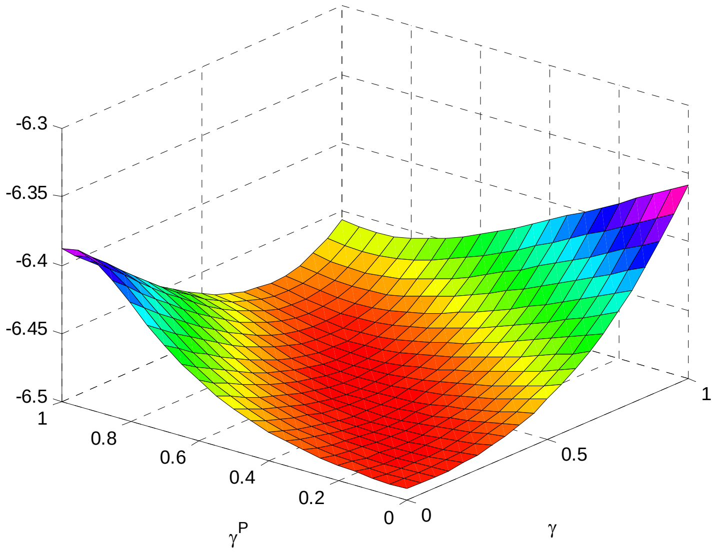

Figure 1: $\ln V(\gamma, \gamma^P, ROR)$

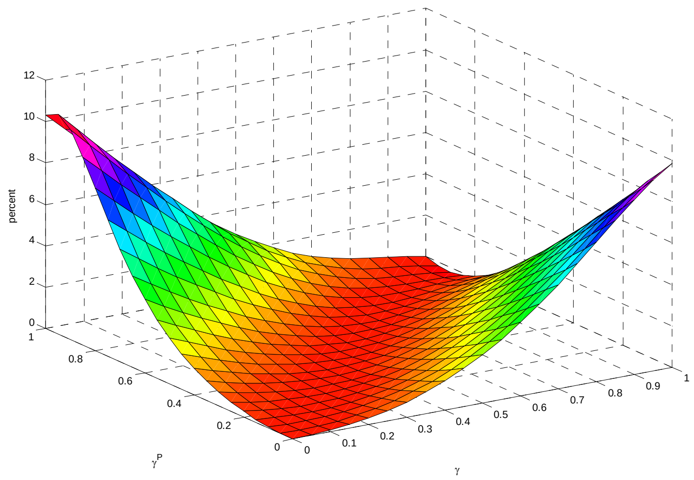

Figure 2: $\mu^*(\gamma, \gamma^P, ROR)$

not allowed to affect the weights in the loss function.[14] Incorporating the dependence of λ_x and λ_i on α will turn out to have important implications for the robustness of ROR policy rules.

I begin, however, by following the standard approach, treating λ_x and λ_i as fixed at their baseline values as α varies. Thus, only the output elasticity of inflation, κ, is allowed to change with α. The ROR rule coefficients are based on the central bank's perceived value α^P while the value in the structural equations is α.

Figure 3 shows log loss as a function of α and α^P. Loss is sensitive to the value of α^P employed in the policy rule, with performance deteriorating by about 600% if the policy maker assumes α is small (little nominal rigidity) when it is in fact large. A policy maker employing a min-max strategy would wish to conduct policy as if α were large. By designing policy as if prices were very sticky (0.7 is the min-max value of α^P), the worst outcome, which occurs when α is actually small, results in only a small deterioration in the loss function.

The experiment lying behind figure 3 assumed the weights in the social loss function were independent of α. However, greater nominal price rigidity makes inflation variability more costly, and therefore inflation stabilization becomes relatively more important. Thus, λ_x (and λ_i) is a decreasing function of α.

The importance of allowing the weights to vary is seen by comparing figure 3, the loss with fixed weights, to figure 4, the loss when the weights λ_x and λ_i are allowed to vary with α. The results change dramatically. Now, overestimating the degree of price stickiness results in a significantly greater deterioration of outcomes then the reverse. The loss from assuming prices are very sticky is large if prices are in fact fairly flexible. The min-max policy rule is the one based on the assumption of fairly flexible prices (a value of 0.05 for α^P).

[14] As noted earlier, exceptions are Kimura and Kurozumi (2003), Kurozumi (2003), and Levin and Williams (2003b).

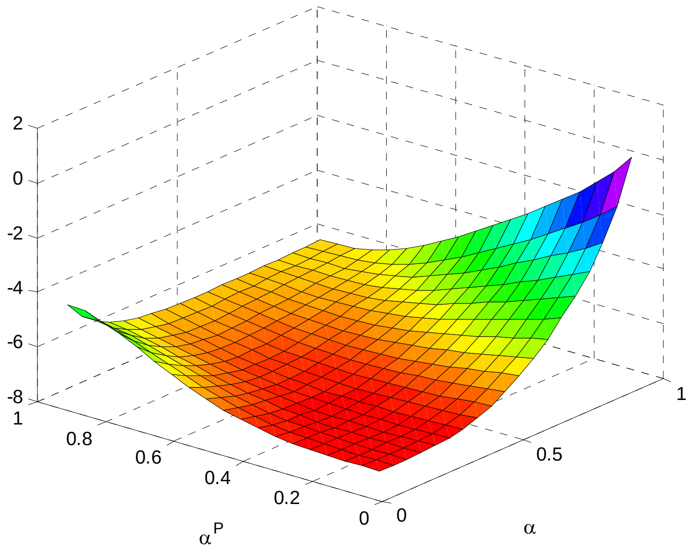

Figure 3: Loss as function of α and α^P, fixed λ_x and λ_i

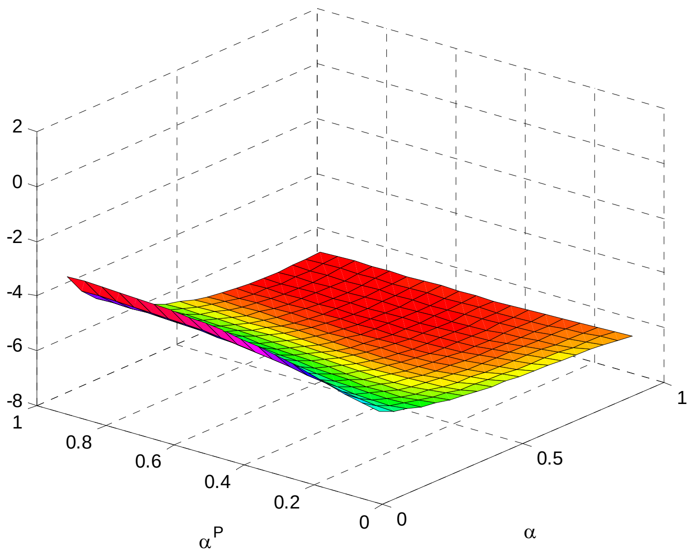

Figure 4: $\ln V(\alpha, \alpha^P, ROR)$

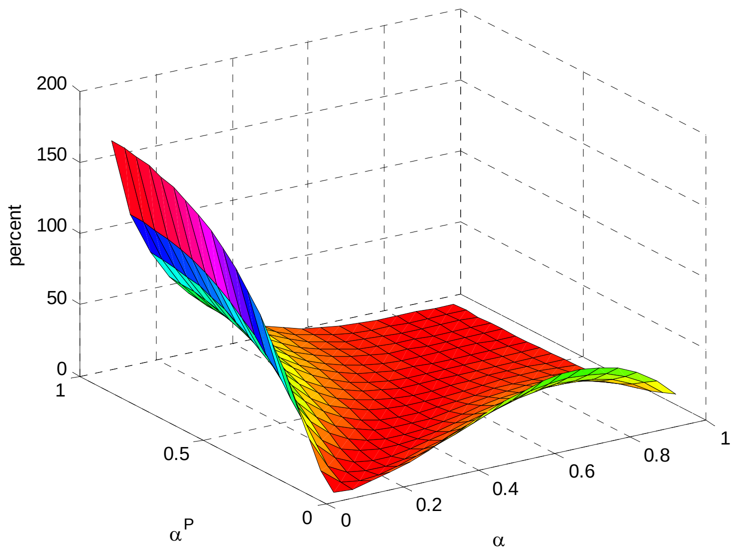

Figure 5: $\mu^*(\alpha, \alpha^P, ROR)$

The reason for the dramatic change in conclusions when the policy weights are endogenized is that both λ_x and λ_i decrease monotonically with α.[15] As a consequence, the policy based on a small α^P is designed to place greater weight on output gap and interest rate stabilization relative to the case when α^P is large. When α is underestimated (so $\alpha^P < \alpha$), policy ends up generating more output gap and interest rate variability then if policy had been based on a correctly specified value of α. With λ_x and λ_i fixed rather than declining with the larger α, this excessive output gap volatility leads to a large deterioration in the loss function.

Figure 5 shows $\mu^*(\alpha, \alpha^P, ROR)$, the percentage loss due to parameter misspecification, when the structural parameters and the loss function weights (κ, λ_x, and λ_i) are all allowed to vary as the degree of price stickiness changes.

[15] λ_x (λ_i) declines from 2.974 (7.7881) when $\alpha = 0.05$ to 0.0001 (0.0014) when $\alpha = 0.95$.

5 The costs of simple rules

The analysis in the previous sections assumed policy was optimal, given the central bank's model specification. Much of the recent literature on monetary policy has instead focused on the implications of simple rules, the Taylor rule being of course the most famous (Taylor 1993). In a simple rule, the policy instrument is adjusted in response to a small number of variables, typically the output gap, inflation, and the lagged nominal interest rate. In optimal simple rules, the response coefficients are chosen to minimize the expected value of the loss function.

When policy is based on a simple rule and there is uncertainty about the true model parameters, macroeconomic outcomes will be inefficient relative to the full information case for two reasons. First, if the simple rule is optimized for parameter values that are in fact incorrect, loss will exceed that achievable under full information. Second, even if the true model parameter values are known, loss under a simple rule will exceed that attainable under the ROR rule. In this section, these two sources of inefficiency are investigated.

5.1 Decomposing the loss function

Let $V(\delta, \delta^P, SR)$ denote the value of the loss function when the actual parameter vector is δ and the central bank implements policy via a simple rule that minimizes the social loss function under the assumption that δ^P is the vector of parameters that characterize the model. When a simple rule is employed and the rule is based on an incorrect estimate of a structural parameter, the resulting loss relative to the loss under the ROR rule with full information is

$$\mu(\delta, \delta^P) \equiv \ln \left[\frac{V(\delta, \delta^P, SR)}{V(\delta, \delta, ROR)} \right].$$

This quantity can be decomposed as

$$\begin{aligned}\mu(\delta,\delta^P) &= \ln\left[\frac{V(\delta,\delta^P,SR)}{V(\delta,\delta,SR)}\right]+\ln\left[\frac{V(\delta,\delta,SR)}{V(\delta,\delta,ROR)}\right]\\ &= \mu^*(\delta,\delta^P,SR)+\theta(\delta,\delta).\end{aligned}$$

In this form, the loss due to misspecification when using a simple rule is equal to the effects of parameter misspecification, given the simple rule is used, plus the inefficiency of the simple rule relative to the ROR rule, given that there is no parameter misspecification. Both $\mu^*(\delta,\delta^P,SR)$ and $\theta(\delta,\delta)$ are non-negative.

An alternative decomposition expresses $\mu(\delta,\delta^P)$ as

$$\begin{aligned}\mu(\delta,\delta^P) &= \ln\left[\frac{V(\delta,\delta^P,SR)}{V(\delta,\delta^P,ROR)}\right]+\ln\left[\frac{V(\delta,\delta^P,ROR)}{V(\delta,\delta,ROR)}\right]\\ &= \theta(\delta,\delta^P)+\mu^*(\delta,\delta^P,ROR),\end{aligned}$$

where $\theta(\delta,\delta^P)$ is the loss in efficiency from using a simple rule rather than the ROR rule, given the misspecification of policy, and $\mu^*(\delta,\delta^P,ROR)$ is the measure previously used to measure the cost of parameter misspecification under the optimal targeting rule. The costs of failing to optimize fully is measured by θ; the costs of failing to estimate correctly is measured by μ^*. Note that while μ and μ^* are both non-negative, θ may be positive or negative. A negative value would indicate that the simple rule is more robust to misspecification than the ROR rule.

The loss under a simple rule will depend on the exact specification of the simple rule. I focus on a class of difference rules of the form

$$i_t = i_{t-1} + a_\pi \pi_t + a_x\left(x_t - x_{t-1}\right). \tag{18}$$

Rules of this form have been shown to perform well in a variety of models (e.g., Orphanides and Williams 2002, Levin, Wieland, and Williams 2003, Levin and Williams 2003a, Walsh 2003c). They are equivalent to a level rule in which the nominal interest rate is adjusted in response to the output gap and the price level. The difference rule imparts an inertia to the interest rate that is absent under a Taylor specification, and this inertia can improve policy trade-offs in forward-looking models (Vestin 2000, Walsh 2003b, Woodford 2003). For each specification of the model, the parameters a_π and a_x are chosen to minimize the unconditional expected value of the loss function given by (5).

5.2 Structural inflation inertia

We have already seen from the plot of $\mu^*(\gamma, \gamma^P, ROR)$ in figure 2 that the costs of misspecifying the degree of inflation inertia are small when the ROR rule is followed. Figure 6 shows the corresponding plot of $\mu^*(\gamma, \gamma^P, SR)$. The vertical scale is the same in the two figures. It is clear that the optimal simple rule is much less sensitive to parameter misspecification than the robustly optimal ROR rule.

Figure 7 shows $\theta(\gamma, \gamma^P)$, the additional (percentage) loss that arises from employing an optimal simple rule of the form (18) rather than the fully optimal ROR rule. Notice that along the diagonal, $\theta(\gamma, \gamma)$ is increasing in γ, indicating that the performance of the simple rule deteriorates, relative to the ROR rule, as inflation becomes more inertial (γ increases). The deterioration rises from 6% to 18% as γ increases from zero to one.

Comparing figures 2 and 7 reveals that the percentage loss associated with the suboptimal rule is generally of the same order of magnitude as the loss due to parameter misspecification under the ROR rule.[16]

An alternative measure of the costs of a simple rule versus the costs of parameter misspecification is given by

$$s(\delta, \delta^P) \equiv 100 \left[\frac{\theta(\delta, \delta^P)}{\mu(\delta, \delta^P)} \right],$$

[16] Recall that the total percentage loss using a simple rule is obtained by adding the surface in figure 7 to the surface in figure 2.

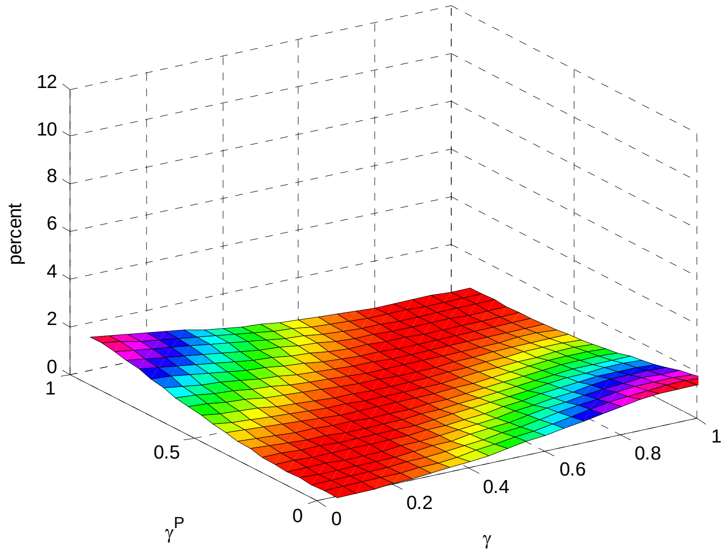

Figure 6: $\mu^*(\gamma, \gamma^P, SR)$

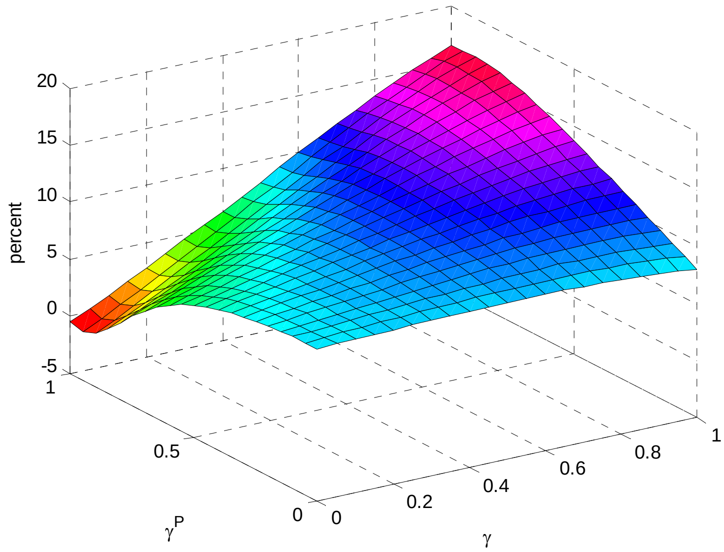

Figure 7: $\theta(\gamma, \gamma^P)$: The efficiency cost of an optimal difference rule relative to the ROR rule

the percentage share of the total inefficiency that is due to using the simple rule. When $\delta^P = \delta$, this measure equals 100%, since the only source of inefficiency is due to the use of the simple rule. As shown in figure 8, the share of the loss attributed to using a simple rule rather than the ROR rule drops off relatively quickly when $\gamma^P > \gamma$ but it declines more gradually when $\gamma^P < \gamma$. However, for most combinations of γ and γ^P, the surface exceeds 50%, implying the inefficiency of the simple rule is greater than that associated with parameter misspecification. An alternative way to assess the efficiency loss due to the simple rule is to calculate the share of the loss given by $s(\gamma, \gamma^P)$ if $\gamma^P = \gamma \pm \varepsilon$ for various values of ε. For example, Galí and Gertler (1999) report two alternative structural estimates of their hybrid inflation model which yield coefficients on lagged inflation that correspond to values for γ of 0.33 and 0.61, a spread of 0.28. Taking ε as 0.3, $s(\gamma, \gamma^P)$ exceeds 88%, suggesting that with respect to structural inflation inertia, it is generally more important to employ an optimal targeting rule than it is to get the exact value of γ correct.

5.3 The degree of price stickiness

Turning now to the effects of misspecifying the value of α under the optimal difference rule, figure 9 shows $\theta(\alpha, \alpha^P)$. Along the diagonal, $\theta(\alpha, \alpha)$ is decreasing in α, indicating that the performance of the simple rule improves, relative to the ROR rule, as prices becomes more sticky (α increases). Comparing this figure to $\mu^*(\alpha, \alpha^P, ROR)$ in figure 5 highlights two conclusions. When policy is based on the assumption of very sticky prices when in fact prices are flexible, the use of the simple rule adds little to the overall costs of misspecification. However, when policy is based on the assumption of very flexible prices when in fact prices are quite sticky, the costs of misspecification are due primarily to the use of a simple rule. Performance deteriorates significantly under a simple rule if the degree of price stickiness is underestimated.

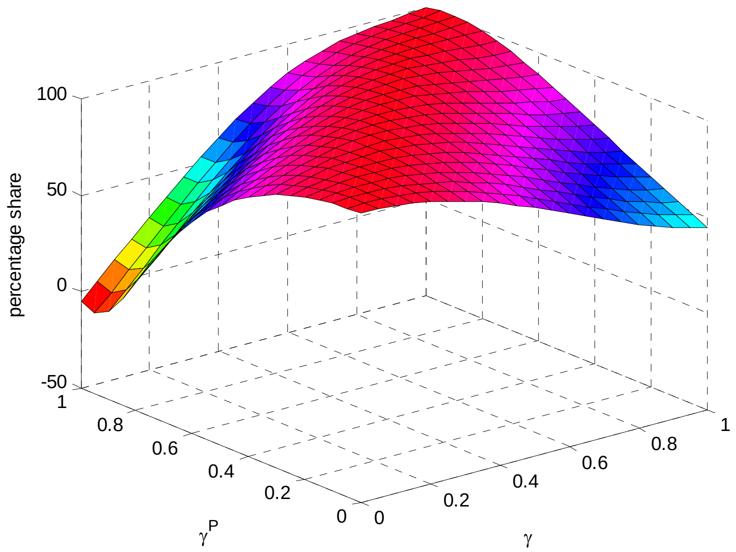

Figure 8: Share of total efficiency cost due to use of simple rule

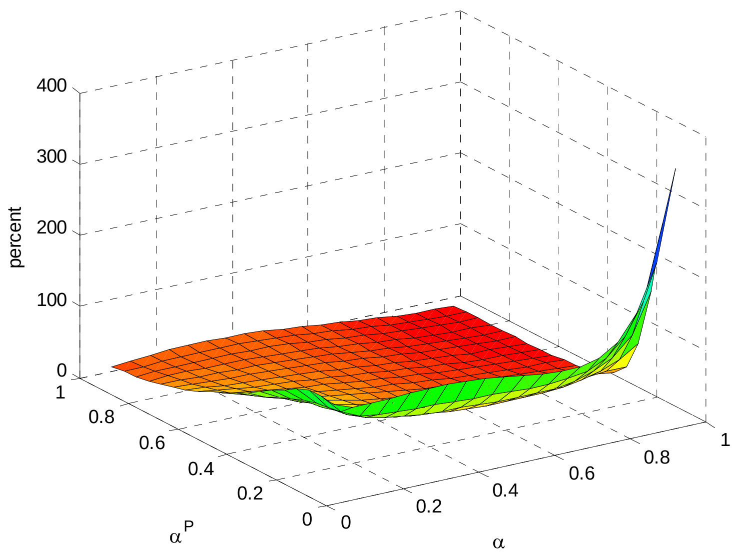

Figure 9: $\theta(\alpha, \alpha^P)$: The efficiency loss from using an optimal difference rule relative to the ROR rule

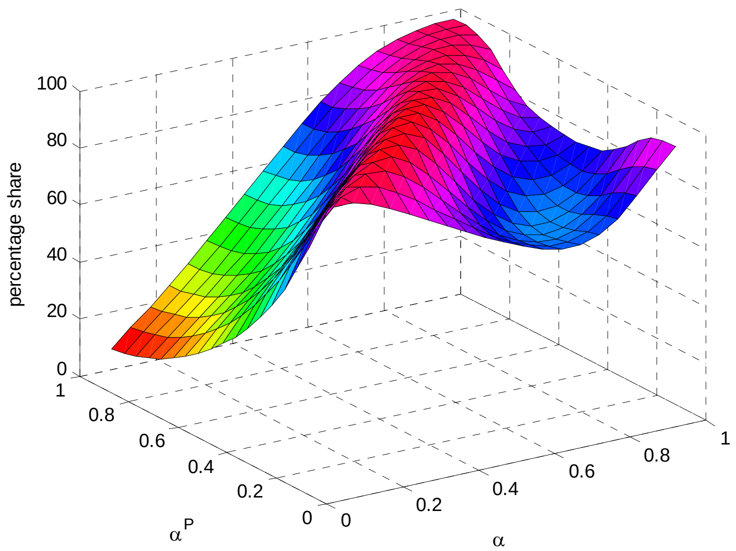

Figure 10: Share of total efficiency cost due to use of optimal difference rule

The share of $\theta(\alpha, \alpha^P)$ in the total loss associated with a simple rule and misspecification of α is shown in figure 10. The sharp peak along the diagonal indicates that the inefficiency due to the simple rule drops off quickly when α is misspecified. However, it is still the case that the surface in figure 10 exceeds 50% for most combinations of α and α^P, again indicating that the gains from employing an optimal rule exceed those of improving the estimate of α.

As was the case with γ, one can calculate the share of the loss given by $s(\alpha, \alpha^P)$ if $\alpha^P = \alpha \pm \varepsilon$. Again taking $\varepsilon = 0.3$[17], $s(\alpha, \alpha^P)$ exceeds 78% for all $\alpha - \varepsilon \leq \alpha^P \leq \alpha$, while $s(\alpha, \alpha^P)$ exceeds 47% for all $\alpha \leq \alpha^P \leq \alpha + \varepsilon$, reflecting the improvement of the simple rule relative to the robustly optimal targeting rule when the central bank over-estimates α.

[17] Eichenbaum and Fischer (2004) report estimates of α that range from 0.56 to 0.90, a spread of 0.34.

6 Summary and conclusions

To investigate the consequences of basing an optimal (explicit) instrument rule on incorrect values of the structural parameters, I employed a calibrated new Keynesian model in which the weights in the social loss function are functions of the model's structural parameters. Thus, employing incorrect parameter values means that the structural equations and the loss function, both of which are important determinants of the targeting rule, will be misspecified.

I focused on two key parameters, the degree of inflation inertia and the degree of price stickiness. In contrast to previous results which find policy outcomes to be sensitive to the degree of inflation inertia, I find that the Giannoni-Woodford optimal rule is robust to misspecification of inflation inertia. In part, this result arises because previous work has ignored the impact of inflation inertia on the social loss function. The rule is not as robust to misspecifying the degree of nominal price stickiness.

Finally, I assess the robustness of optimal difference rules as an example of a simple policy rule that has previously been shown to perform well. For uncertainty about the degree of inflation inertia, the simple rule was more robust than the optimal targeting rule. However, for most combinations of actual and perceived structural inflation inertia, the loss from using a simple rule rather than the optimal targeting rule exceeded the costs arising from parameter misspecification. When the degree of nominal rigidity is potentially misspecified, the simple rule's performance deteriorated significantly if the policy maker thinks prices are relatively flexible when in fact they are quite sticky. For most combinations of actual and perceived price stickiness, however, the loss from using a simple rule rather than the optimal targeting rule exceeded the costs arising from parameter misspecification.

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Appendix: Robustly optimal rules

The first order conditions (8) and (10) imply,

$$s_{2t+1} = -\sigma\lambda_i(i_t - i^*) \tag{19}$$

and

$$s_{1t+1} = \left(\frac{\lambda_x}{\kappa}\right) x_t - \left(\frac{\sigma\lambda_i}{\kappa}\right)\left(i_t - \beta^{-1} i_{t-1}\right) + \left(\frac{\sigma\lambda_i}{\kappa}\right)\left(1 - \beta^{-1}\right) i^*, \tag{20}$$

while (9) implies

$$s_{3t+1} = \left(\frac{\lambda_i}{\beta}\right)(i_{t-1} - i^*) + \beta\gamma E_t s_{3t+2}. \tag{21}$$

Letting Δ denote the first difference operator, (7) becomes $s_{3t+1} = -\left(z_t + \Delta s_{1t+1}\right)$, which with (21) gives

$$z_t + \Delta s_{1t+1} = -\left(\frac{\lambda_i}{\beta}\right)(i_{t-1} - i^*) + \beta\gamma E_t\left(z_{t+1} + \Delta s_{1t+2}\right). \tag{22}$$

First differencing (20) to eliminate Δs_{1t+1} and $E_t \Delta s_{1t+2}$ in (22) yields the robustly optimal targeting rule given by (11), where $q_t \equiv z_t + (\lambda_x/\kappa)\Delta x_t$.

To obtain a targeting rule that does not involve expectations of future interest rates, inflation, or the output gap, use (20) in (7) to yield

$$q_t - \left(\frac{\sigma\lambda_i}{\kappa}\right)\Delta i_t + \left(\frac{\sigma\lambda_i}{\beta\kappa}\right)\Delta i_{t-1} + s_{3t+1} = 0. \tag{23}$$

Combining this with (21) yields

$$q_t - \left(\frac{\sigma\lambda_i}{\kappa}\right)\Delta i_t + \left(\frac{\sigma\lambda_i}{\beta\kappa}\right)\Delta i_{t-1} + \left(\frac{\lambda_i}{\beta}\right)(i_{t-1} - i^*) + \beta\gamma E_t s_{3t+2} = 0.$$

Solving for i_t gives a policy rule in which expectations of the future appear only through the term involving s_{3t+2}:

$$i_t = -\left(\frac{\kappa}{\sigma\beta}\right) i^* + \left(\frac{\kappa}{\sigma\lambda_i}\right) q_t + \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta\sigma}\right) i_{t-1} - \left(\frac{1}{\beta}\right) i_{t-2} + \left(\frac{\beta\gamma\kappa}{\sigma\lambda_i}\right) E_t s_{3t+2}.$$

In the rational expectations solution,

$$s_{3t} = C_{31}Y_t + C_{32}\phi_{2t}$$

where $\phi_{2t} = [s_{1t}\ s_{2t}]'$ and

$$\begin{bmatrix} Y_{t+1} \\ \phi_{2t+1} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ \phi_{2t} \end{bmatrix}.$$

Thus,

$$\begin{aligned} E_t s_{3t+2} &= C_{31}E_tY_{t+2} + C_{32}E_t\phi_{2t+2} \\ &= \left[C_{31}\left(M_{11}^2 + M_{12}M_{21}\right) + C_{32}\left(M_{21}M_{11} + M_{22}M_{21}\right)\right]Y_t \\ &\quad + \left[C_{31}\left(M_{11}M_{12} + M_{12}M_{22}\right) + C_{32}\left(M_{21}M_{12} + M_{22}^2\right)\right]\phi_{2t} \\ &= C_{31}^*Y_t + C_{32}^*\phi_{2t}, \end{aligned}$$

where

$$C_{31}^* = \left[C_{31}\left(M_{11}^2 + M_{12}M_{21}\right) + C_{32}\left(M_{21}M_{11} + M_{22}M_{21}\right)\right]$$

and

$$C_{32}^* = \left[C_{31}\left(M_{11}M_{12} + M_{12}M_{22}\right) + C_{32}\left(M_{21}M_{12} + M_{22}^2\right)\right].$$

Hence,

$$\begin{aligned} i_t &= -\left(\frac{\kappa}{\sigma\beta}\right)i^* + \left(\frac{\kappa}{\sigma\lambda_i}\right)q_t + \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta\sigma}\right)i_{t-1} - \left(\frac{1}{\beta}\right)i_{t-2} \\ &\quad + \left(\frac{\beta\gamma\kappa}{\sigma\lambda_i}\right)\left(C_{31}^*Y_t + C_{32}^*\phi_{2t}\right). \end{aligned}$$

and

$$\phi_{2t+1} = M_{21}Y_t + M_{22}\phi_{2t}$$

represent a system in i_t and ϕ_{2t+1} that the central bank can solve for its optimal interest rate setting that does not involve forecasts of endogenous variables.

To implement this approach, the model is first solved under the assumption that the policy rule is based on the correct values of the structural parameters. This solution provides the values of the $M_{ij}(\delta)$ and $C^*_{3j}(\delta)$ matrices, where δ denotes the vector of parameter values. When policy is based on incorrect parameters, then the structural equations based on δ are combined with

$$\begin{aligned} i_t \quad = \quad & -\left(\frac{\kappa^P}{\sigma\beta}\right) i^* + \left(\frac{\kappa^P}{\sigma\lambda_i^P}\right)\left[z_t^P + \left(\frac{\lambda_x}{\kappa}\right)\Delta y_t\right] + \left(1 + \frac{1}{\beta} + \frac{\kappa^P}{\beta\sigma}\right) i_{t-1} \\ & -\left(\frac{1}{\beta}\right) i_{t-2} + \left(\frac{\beta\gamma^P\kappa^P}{\sigma\lambda_i^P}\right)\left(C^*_{31}(\delta^P)Y_t + C^*_{31}(\delta^P)\phi_{2t}\right). \end{aligned}$$

and

$$\phi_{2t+1} = M^P_{21}Y_t + M^P_{22}\phi_{2t},$$

where

$$z_t^P = z_t + (\gamma - \gamma^P)\pi_{t-1},$$

and the resulting system is solved for the equilibrium. In this setup, s_{1t} and s_{2t}, the elements of ϕ_{2t}, are added to the state vector so that the model becomes

$$\hat{A}\begin{bmatrix} \hat{Y}_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \hat{B}\begin{bmatrix} \hat{Y}_t \\ y_t \end{bmatrix} + \hat{G}u_t + \begin{bmatrix} \hat{\psi}_{t+1} \\ 0 \end{bmatrix}$$

where $\hat{Y}_t = [s_{1t}\ s_{2t}\ e_t\ r_t^n\ i_{t-2}\ i_{t-1}\ \pi_{t-1}\ x_{t-1}]'$ is an expanded list of predetermined variables and $y_t = [z_t\ x_t]'$.

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